

Decay constants and form factors of s -wave and p -wave mesons in the covariant light-front quark model

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Abstract

We reanalyze the decay constants of s -wave and p -wave mesons and $D, B \rightarrow M$ form factors, where M represents a pseudoscalar meson, a vector meson, a scalar meson, or an axial vector meson within a covariant light-front quark model. The parameter β for wave-functions of most of s -wave mesons and of a few axial-vector mesons are fixed with latest experimental information, wherever available or using the lattice calculations. The treatment of masses and mixing angles for strange axial vector mesons is improved for the purpose. We extend our analysis to determine the form factors appearing in the transition of $D_s, B_s \rightarrow M$ transitions, and to the isoscalar final state mesons. Numerical results of the form factors for transitions between a heavy pseudoscalar meson and an s -wave or p -wave light meson and their momentum dependence are presented in detail. Further, their sensitivity to uncertainties of β parameters of the initial as well as the final mesons is investigated. Some experimental measurements of the charmed and bottom meson decays are employed to compare the decay constants and transition form factors obtained in this and other works.

I. INTRODUCTION

In the previous work [1], various $P \rightarrow M$ form factors, where P represents a heavy pseudoscalar meson (D or B), and M represents either s -wave or low-lying p -wave meson, were calculated within the framework of the covariant light-front (CLF) approach. This formalism preserves the Lorentz covariance in the light-front framework and has been applied successfully to describe various properties of pseudoscalar and vector mesons [2–4]. The analysis of the covariant light-front quark model to transitions of the charmed and bottom mesons was extended to even parity, p -wave mesons [1]. Recently, the CLF approach has also been used to the studies of the quarkonia [5, 6], the p -wave meson emitting decays of the bottom mesons [7] and the B_c system [8] and so on. In the present work, we update our results for D and B meson form factors, and extend this analysis to determine the form factors appearing in the $D_s, B_s \rightarrow M$ transitions, and to the flavor-diagonal final state mesons M . Experimental measurements of the decays of the τ lepton, pseudoscalar and vector mesons are employed to determine the decay constants, which in turn fix the shape parameters, β , of the respective mesons. For a few cases, the decay constants estimated by lattice calculations have been used for this purpose. We have now used the improved estimation of the K_{1A} and K_{1B} mixing angle, where K_{1A} and K_{1B} are the 3P_1 and 1P_1 states of K_1 , respectively, which are related to the physical $K_1(1270)$ and $K_1(1400)$ states.

We then study transitions of the heavy flavor pseudoscalar mesons to pseudoscalar mesons (P), vector mesons (V), scalar mesons (S) and axial vector mesons (A) within the CLF model. Numerical results of the form factors for these transitions and their momentum dependence are presented in detail. In particular, all the form factors for heavy-to-light and heavy-to-heavy transitions for charmed mesons (D, D_s) and bottom mesons (B, B_s) are calculated. Further, their sensitivity to uncertainties of β parameters of the initial as well as of the final mesons is investigated separately. Theoretically, the Isgur-Scora-Grinstein-Wise (ISGW) quark model [9, 10] has been the only model for a long time that could provide a systematical estimate of the transition of a ground-state s -wave meson to a low-lying p -wave meson. However, this model is based on the nonrelativistic constituent quark picture. We have earlier pointed out [1] that relativistic effects could manifest in heavy-to-light transitions at maximum recoil where the final-state meson can be highly relativistic. For example, the $B \rightarrow a_1$ form factor $V_0^{Ba_1}(0)$ is found to be 0.13 in the relativistic light-front model [1], while it is as big as 1.01 in the ISGW model [9]. Hence there is no reason to expect that the nonrelativistic quark model is still applicable there, though in the improved version of the model (ISGW2) [11] a number of improvements, such as the constraints imposed by heavy quark symmetry and hyperfine distortions of wave functions have been incorporated. We believe that the CLF quark model can provide useful and reliable information on $B \rightarrow M$ transitions particularly at maximum recoil.

The paper is organized as follows. The basic features of the covariant light-front (CLF) model are recapitulated in Sec. II. In Sec. III, decay constants are presented in the CLF model. Available experimental measurements for various decays are used to determine decay constants, which in turn are used to fix β parameters of the CLF model. Sometimes, lattice predictions for few decay constants are also used for this purpose. In Sec. IV, the analysis of form factors appearing for

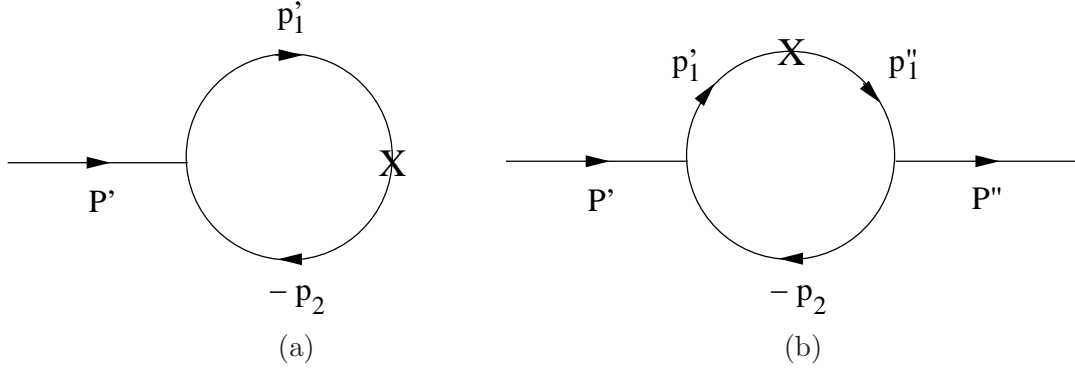


FIG. 1: Feynman diagrams for (a) meson decay and (b) meson transition amplitudes, where $P'^{(n)}$ is the incoming (outgoing) meson momentum, $p_1'^{(n)}$ is the quark momentum, p_2 is the anti-quark momentum and X denotes the corresponding $V - A$ current vertex.

transitions from pseudoscalar mesons to s -wave mesons (pseudoscalar or vector) and p -wave mesons (scalar and axial vector) is given. In Sec. V, numerical results are presented for these form factors and their q^2 - dependence taking proper inclusions of uncertainties in the shape parameter, β . Summary and conclusions are given in Sec. VI.

II. FORMALISM OF A COVARIANT LIGHT-FRONT MODEL

In the conventional light-front framework, the constituent quarks of the meson are required to be on their mass shells and various physical quantities are extracted from the plus component of the corresponding current matrix elements. However, this procedure will miss the zero-mode effects and render the matrix elements non-covariant. Jaus [2, 3] has proposed a covariant light-front approach that permits a systematical way of dealing with the zero mode contributions. Physical quantities such as the decay constants and form factors can be calculated in terms of Feynman momentum loop integrals which are manifestly covariant. This of course means that the constituent quarks of the bound state are off-shell. In principle, this covariant approach will be useful if the vertex functions can be determined by solving the QCD bound state equation. In practice, we would have to be contented with the phenomenological vertex functions such as those employed in the conventional light-front model. Therefore, using the light-front decomposition of the Feynman loop momentum, say p_μ , and integrating out the minus component of the loop momentum p^- , one goes from the covariant calculation to the light-front one. Moreover, the antiquark is forced to be on its mass shell after p^- integration. Consequently, one can replace the covariant vertex functions by the phenomenological light-front ones.

To begin with, we consider decay and transition amplitudes given by one-loop diagrams as shown in Fig. 1 for the decay constants and form factors of ground-state s -wave mesons and low-lying p -wave mesons. We follow the approach of [1, 4] and use the same notation. The incoming (outgoing) meson has the momentum $P'^{(n)} = p_1'^{(n)} + p_2$, where $p_1'^{(n)}$ and p_2 are the momenta of the off-shell

quark and antiquark, respectively, with masses $m_1^{(')}$ and m_2 . These momenta can be expressed in terms of the internal variables (x_i, p'_\perp) ,

$$p'_{1,2}{}^+ = x_{1,2}P'^+, \quad p'_{1,2\perp} = x_{1,2}P'_\perp \pm p'_\perp, \quad (2.1)$$

with $x_1 + x_2 = 1$. Note that we use $P' = (P'^+, P'^-, P'_\perp)$, where $P'^\pm = P'^0 \pm P'^3$, so that $P'^2 = P'^+P'^- - P'^2_\perp$.

In the covariant light-front approach, total four momentum is conserved at each vertex where quarks and antiquarks are off-shell. These differ from the conventional light-front approach (see, for example [3, 12]) where the plus and transverse components of momentum are conserved, and quarks as well as antiquarks are on-shell.

It is useful to define some internal quantities for on-shell quarks:

$$\begin{aligned} M_0'^2 &= (e'_1 + e_2)^2 = \frac{p_\perp'^2 + m_1'^2}{x_1} + \frac{p_\perp'^2 + m_2^2}{x_2}, & \widetilde{M}_0' &= \sqrt{M_0'^2 - (m'_1 - m_2)^2}, \\ e_i^{(')} &= \sqrt{m_i^{(')2} + p_\perp'^2 + p_z'^2}, & p_z' &= \frac{x_2 M_0'}{2} - \frac{m_2^2 + p_\perp'^2}{2x_2 M_0'}. \end{aligned} \quad (2.2)$$

Here $M_0'^2$ can be interpreted as the kinetic invariant mass squared of the incoming $q\bar{q}$ system, and e_i the energy of the quark i .

It has been shown in [13] that one can pass to the light-front approach by integrating out the p^- component of the internal momentum in covariant Feynman momentum loop integrals. We need Feynman rules for the meson-quark-antiquark vertices to calculate the amplitudes shown in Fig. 1. These Feynman rules for vertices ($i\Gamma'_M$) of ground-state s -wave mesons and low-lying p -wave mesons are summarized in Table I. Next, we shall find the decay constants in the covariant light-front approach.

TABLE I: Feynman rules for the vertices ($i\Gamma'_M$) of the incoming mesons-quark-antiquark, where p'_1 and p_2 are the quark and antiquark momenta, respectively. Under the contour integrals to be discussed below, H'_M and W'_M are reduced to h'_M and w'_M , respectively, whose expressions are given by Eq. (3.10). Note that for outgoing mesons, we shall use $i(\gamma_0\Gamma_M^\dagger\gamma_0)$ for the corresponding vertices.

$M(^{2S+1}L_J)$	$i\Gamma'_M$
pseudoscalar (1S_0)	$H'_P\gamma_5$
vector (3S_1)	$iH'_V[\gamma_\mu - \frac{1}{W'_V}(p'_1 - p_2)_\mu]$
scalar (3P_0)	$-iH'_S$
axial (3P_1)	$-iH'_{3A}[\gamma_\mu + \frac{1}{W'_{3A}}(p'_1 - p_2)_\mu]\gamma_5$
axial (1P_1)	$-iH'_{1A}[\frac{1}{W'_{1A}}(p'_1 - p_2)_\mu]\gamma_5$

III. DECAY CONSTANTS

The decay constants for $J = 0, 1$ mesons are defined by the matrix elements

$$\begin{aligned}\langle 0|A_\mu|P(P')\rangle &\equiv \mathcal{A}_\mu^P = if_P P'_\mu, & \langle 0|V_\mu|S(P')\rangle &\equiv \mathcal{A}_\mu^S = f_S P'_\mu, \\ \langle 0|V_\mu|V(P', \varepsilon')\rangle &\equiv \mathcal{A}_\mu^V = M'_V f_V \varepsilon'_\mu, & \langle 0|A_\mu|^3(1)A(P', \varepsilon')\rangle &\equiv \mathcal{A}_\mu^{3A(1A)} = M'_{3A(1A)} f_{3A(1A)} \varepsilon'_\mu,\end{aligned}\quad (3.1)$$

where the $^{2S+1}L_J = ^1S_0, ^3P_0, ^3S_1, ^3P_1$ and 1P_1 states of $q_1\bar{q}_2$ mesons are denoted by $P, S, V, ^3A$ and 1A , respectively. It is useful to note that in the $SU(N)$ -flavor limit ($m'_1 = m_2$) we should have vanishing f_S and f_{1A} . The former can be seen by applying equations of motion to the matrix element of the scalar resonance in Eq. (3.1) to obtain

$$m_S^2 f_S = i(m'_1 - m_2)\langle 0|\bar{q}_1 q_2|S\rangle. \quad (3.2)$$

The latter is based on the argument that the light 3P_1 and 1P_1 states transfer under charge conjugation as

$$M_a^b(^3P_1) \rightarrow M_b^a(^3P_1), \quad M_a^b(^1P_1) \rightarrow -M_b^a(^1P_1), \quad (a = 1, 2, 3), \quad (3.3)$$

where the light axial-vector mesons are represented by a 3×3 matrix. Since the weak axial-vector current transfers as $(A_\mu)_a^b \rightarrow (A_\mu)_b^a$ under charge conjugation, it is clear that the decay constant of the 1P_1 meson vanishes in the $SU(3)$ limit [14]. This argument can be generalized to heavy axial-vector mesons. In fact, under similar charge conjugation argument $[(V_\mu)_a^b \rightarrow -(V_\mu)_b^a, M_a^b(^3P_0) \rightarrow M_b^a(^3P_0)]$ one can also prove the vanishing of f_S in the $SU(N)$ limit.

Furthermore, in the heavy quark limit ($m'_1 \rightarrow \infty$), the heavy quark spin s_Q decouples from the other degrees of freedom so that s_Q and the total angular momentum of the light antiquark j are separately good quantum numbers. Hence, it is more convenient to use the $L_J^j = P_2^{3/2}, P_1^{3/2}, P_1^{1/2}$ and $P_0^{1/2}$ basis. It is obvious that the first and the last of these states are 3P_2 and 3P_0 , respectively, while [15]

$$|P_1^{3/2}\rangle = \sqrt{\frac{2}{3}}|^1P_1\rangle + \frac{1}{\sqrt{3}}|^3P_1\rangle, \quad |P_1^{1/2}\rangle = \frac{1}{\sqrt{3}}|^1P_1\rangle - \sqrt{\frac{2}{3}}|^3P_1\rangle. \quad (3.4)$$

Heavy quark symmetry (HQS) requires [10, 16]

$$f_V = f_P, \quad f_{A^{1/2}} = f_S, \quad f_{A^{3/2}} = 0, \quad (3.5)$$

where we have denoted the $P_1^{1/2}$ and $P_1^{3/2}$ states by $A^{1/2}$ and $A^{3/2}$, respectively. These relations in the above equation can be understood from the fact that $(S_0^{1/2}, S_1^{1/2}), (P_0^{1/2}, P_1^{1/2})$ and $(P_1^{3/2}, P_2^{3/2})$ form three doublets in the HQ limit and that the tensor meson cannot be induced from the $V - A$ current.

Following the procedure described in [1, 4], we now evaluate meson decay constants through the following formulas:

$$f_P = \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_P}{x_1 x_2 (M'^2 - M_0'^2)} 4(m'_1 x_2 + m_2 x_1), \quad (3.6)$$

$$f_V = \frac{N_c}{4\pi^3 M'} \int dx_2 d^2 p'_\perp \frac{h'_V}{x_1 x_2 (M'^2 - M_0'^2)} \times \left[x_1 M_0'^2 - m'_1(m'_1 - m_2) - p_\perp'^2 + \frac{m'_1 + m_2}{w'_V} p_\perp'^2 \right], \quad (3.7)$$

$$f_S = \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_S}{x_1 x_2 (M'^2 - M_0'^2)} 4(m'_1 x_2 - m_2 x_1), \quad (3.8)$$

$$f_{3A} = -\frac{N_c}{4\pi^3 M'} \int dx_2 d^2 p'_\perp \frac{h'_{3A}}{x_1 x_2 (M'^2 - M_0'^2)} \times \left[x_1 M_0'^2 - m'_1(m'_1 + m_2) - p_\perp'^2 - \frac{m'_1 - m_2}{w'_{3A}} p_\perp'^2 \right],$$

$$f_{1A} = \frac{N_c}{4\pi^3 M'} \int dx_2 d^2 p'_\perp \frac{h'_{1A}}{x_1 x_2 (M'^2 - M_0'^2)} \left(\frac{m'_1 - m_2}{w'_{1A}} p_\perp'^2 \right), \quad (3.9)$$

where

$$h'_P = h'_V = (M'^2 - M_0'^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2} \widetilde{M}_0'} \varphi',$$

$$h'_S = \sqrt{\frac{2}{3}} h'_{3A} = (M'^2 - M_0'^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2} \widetilde{M}_0'} \frac{\widetilde{M}_0'^2}{2\sqrt{3} M_0'} \varphi'_p,$$

$$h'_{1A} = h'_T = (M'^2 - M_0'^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2} \widetilde{M}_0'} \varphi'_p,$$

$$w'_V = M_0' + m'_1 + m_2, \quad w'_{3A} = \frac{\widetilde{M}_0'^2}{m'_1 - m_2}, \quad w'_{1A} = 2, \quad (3.10)$$

are the appropriate replacements of the vertex functions,

$$H'_M \rightarrow \hat{H}'_M = H'_M(\hat{p}_1'^2, \hat{p}_2'^2) \equiv h'_M,$$

$$W'_M \rightarrow \hat{W}'_M = W'_M(\hat{p}_1'^2, \hat{p}_2'^2) \equiv w'_M, \quad (3.11)$$

appearing in the matrix elements of annihilation of a meson state via weak currents, and φ' and φ'_p are the light-front momentum distribution amplitudes for s -wave and p -wave mesons, respectively. There are several popular phenomenological light-front wave functions that have been employed to describe various hadronic structures in the literature. In the present work, we shall use the Gaussian-type wave function [17]

$$\varphi' = \varphi'(x_2, p'_\perp) = 4 \left(\frac{\pi}{\beta'^2} \right)^{\frac{3}{4}} \sqrt{\frac{dp'_z}{dx_2}} \exp \left(-\frac{p_z'^2 + p_\perp'^2}{2\beta'^2} \right),$$

$$\varphi'_p = \varphi'_p(x_2, p'_\perp) = \sqrt{\frac{2}{\beta'^2}} \varphi', \quad \frac{dp'_z}{dx_2} = \frac{e'_1 e_2}{x_1 x_2 M_0'}. \quad (3.12)$$

The parameter β' , which describes the momentum distribution, is expected to be of order Λ_{QCD} .

Note that with the explicit form of h'_P shown in Eq. (3.10), the familiar expression of f_P in the conventional light-front approach [3, 12], namely,

$$f_P = 2 \frac{\sqrt{2N_c}}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{1}{\sqrt{x_1 x_2} \widetilde{M}'_0} (m'_1 x_2 + m_2 x_1) \varphi'(x_2, p'_\perp), \quad (3.13)$$

is reproduced. For decay constants of vector and axial-vector mesons, we consider the case with the transverse polarization given by

$$\varepsilon(\pm) = \left(\frac{2}{P'^+} \varepsilon_\perp \cdot P'_\perp, 0, \varepsilon_\perp \right), \quad \varepsilon_\perp = \mp \frac{1}{\sqrt{2}} (1, \pm i). \quad (3.14)$$

For $m'_1 = m_2$, the meson wave function is symmetric with respect to x_1 and x_2 , and hence $f_S = 0$, as it should be. Similarly, it is clear that $f_{1A} = 0$ for $m'_1 = m_2$. The SU(N)-flavor constraints on f_S and f_{1A} are thus satisfied.¹

To perform numerical computations of decay constants and form factors, we need to specify the input parameters in the covariant light front model. These are the constituent quark masses and the shape parameter β appearing in the Gaussian-type wave function (3.12). For constituent quark masses, we use [1, 4, 12, 18, 19]

$$m_{u,d} = 0.26 \text{ GeV}, \quad m_s = 0.45 \text{ GeV}, \quad m_c = 1.40 \text{ GeV}, \quad m_b = 4.64 \text{ GeV}. \quad (3.15)$$

Shown in Tables II and III are the input parameter β and decay constants, respectively. In Table III the decay constants in parentheses are used to determine β using the analytic expressions in the covariant light-front model as given above. For most of s -wave mesons, and a few axial

TABLE II: The input parameter β (in units of GeV) in the Gaussian-type wave function (3.12) for mesons. Note that $\beta_{q\bar{q}}$ is used for the $(u\bar{u} + d\bar{d})/\sqrt{2}$ state.

$^{2S+1}L_J$	1S_0	3S_1	3P_0	3P_1	1P_1
$\beta_{d\bar{u}}$	$0.3077^{+0.0009}_{-0.0008}$	$0.2815^{+0.0046}_{-0.0047}$	$0.2983^{+0.0123}_{-0.0129}$	$0.2983^{+0.0123}_{-0.0129}$	$0.2983^{+0.0123}_{-0.0129}$
$\beta_{q\bar{q}}$	$0.3499^{+0.0136}_{-0.0129}$	$0.2640^{+0.0031}_{-0.0032}$	0.2983 ± 0.0298	0.2983 ± 0.0298	0.2983 ± 0.0298
$\beta_{s\bar{u}}$	$0.3479^{+0.0029}_{-0.0029}$	$0.2926^{+0.0047}_{-0.0047}$	$0.3224^{+0.0163}_{-0.0195}$	$0.3224^{+0.0163}_{-0.0195}$	$0.3224^{+0.0163}_{-0.0195}$
$\beta_{s\bar{s}}$	$0.3598^{+0.0220}_{-0.0208}$	0.3083 ± 0.0014	0.3492 ± 0.0064	0.3492 ± 0.0064	0.3492 ± 0.0064
$\beta_{c\bar{u}}$	$0.4656^{+0.0217}_{-0.0212}$	0.4255 ± 0.0426	0.3890 ± 0.0389	0.3890 ± 0.0389	0.3890 ± 0.0389
$\beta_{c\bar{s}}$	$0.5358^{+0.0137}_{-0.0135}$	0.4484 ± 0.0448	0.3900 ± 0.0390	0.3900 ± 0.0390	0.3900 ± 0.0390
$\beta_{c\bar{c}}$	$0.7690^{+0.0049}_{-0.0049}$	0.6492 ± 0.0069	0.4200 ± 0.0420	0.4200 ± 0.0420	0.4200 ± 0.0420
$\beta_{b\bar{u}}$	$0.5547^{+0.0260}_{-0.0261}$	0.5183 ± 0.0518	0.5000 ± 0.0500	0.5000 ± 0.0500	0.5000 ± 0.0500
$\beta_{b\bar{s}}$	$0.6103^{+0.0330}_{-0.0331}$	0.5589 ± 0.0559	0.5500 ± 0.0550	0.5500 ± 0.0550	0.5500 ± 0.0550
$\beta_{b\bar{c}}$	0.9582 ± 0.0958	0.8451 ± 0.0845	0.6800 ± 0.0680	0.6800 ± 0.0680	0.6800 ± 0.0680
$\beta_{b\bar{b}}$	1.4514 ± 0.0132	1.3267 ± 0.0100	0.9993 ± 0.0999	0.9993 ± 0.0999	0.9993 ± 0.0999

¹ We wish to stress that the vector decay constant obtained in the conventional light-front model [3] does not coincide with the above result (3.7) owing to the missing zero mode contribution.

TABLE III: Meson decay constants (in units of MeV) obtained by using Eqs. (3.6), (3.8), (3.7) and (3.9). Those in parentheses are taken as inputs to determine the corresponding β 's shown in Table II. Decay constants of some p -wave mesons are also used as inputs (see the text for details). Here $f_{q\bar{q}}$ denotes decay constant for the $(u\bar{u} + d\bar{d})/\sqrt{2}$ state.

${}^{2S+1}L_J$	1S_0	3S_1	3P_0	3P_1	1P_1
$f_{d\bar{u}}$	(130.41 ± 0.20)	(215 ± 5)	0	(-203 ∓ 18)	0
$f_{q\bar{q}}$	(139.54 ± 2.62)	(195 ± 3)	0	-193_{+38}^{-43}	0
$f_{s\bar{u}}$	(156.1 ± 0.9)	(217 ± 5)	$34.9_{-1.8}^{+1.4}$	(-212_{+26}^{-23})	$20.4_{-1.8}^{+1.5}$
$f_{s\bar{s}}$	(174.75 ± 7.83)	(228 ± 2)	0	(-230 ∓ 9)	0
$f_{c\bar{u}}$	(206.7 ± 8.9)	$(245)_{-34}^{+35}$	107 ± 13	-177_{+34}^{-38}	$59.6_{-9.5}^{+9.8}$
$f_{c\bar{s}}$	(254.6 ± 5.9)	$(272)_{-38}^{+39}$	$74.4_{-10.6}^{+10.4}$	-159_{+32}^{-36}	$42.2_{-7.3}^{+7.6}$
$f_{c\bar{c}}$	(394.7 ± 2.4)	(411 ± 6)	0	-105_{+23}^{-26}	0
$f_{b\bar{u}}$	(193 ± 11)	$(196)_{-27}^{+28}$	143 ± 21	-155_{+28}^{-30}	$83.6_{-13.6}^{+14.3}$
$f_{b\bar{s}}$	(231 ± 15)	$(229)_{-31}^{+32}$	139 ± 22	-166_{+31}^{-34}	$82.6_{-14.2}^{+15.0}$
$f_{b\bar{c}}$	440_{-52}^{+51}	440_{-52}^{+51}	$90.6_{-16.6}^{+17.5}$	-155_{+33}^{-37}	$52.0_{-10.3}^{+11.2}$
$f_{b\bar{b}}$	(708 ± 8)	(708 ± 8)	0	-185_{+42}^{-48}	0

vector mesons, these are fixed from the latest decay rates given in the Particle Data Group [20], or other analysis based on some experimental results. For decay constants of some heavy flavor mesons, we have used recent lattice results to fix β . For the remaining p -wave mesons, we use the β parameters obtained in the ISGW2 model [11], the improved version of the ISGW model, up to some simple scaling. In this paper, we have investigated the variation of the form factors and their slope parameters for q^2 dependence with the variation of β values. Wherever the experimental information is available, we have used that to fix the errors for the corresponding β values, otherwise arbitrarily introduced an uncertainty of 10% in β for some s -wave and p -wave mesons.

Several remarks are in order:

(i) Decay constants of the charged pseudoscalar mesons, π^+ , K^+ , D^+ , D_s^+ , and B^- (and their charge-conjugate partners) can be determined from their purely leptonic decay rates. These mesons formed from a quark and anti-quark can decay to a charged lepton pair when their constituents annihilate via a virtual W boson. Now quite precise measurements are available for the branching fractions of $P \rightarrow \ell \nu_\ell$ decays [20]. Following the analysis of Rosner and Stone [21] for the available branching fractions, we take $f_\pi = 130.41 \pm 0.20$, $f_K = 156.10 \pm 0.85$, $f_D = 206.7 \pm 8.9$ (all in MeV) to fix the β parameters of the respective mesons.

(ii) For fixing β_{D_s} , we have taken the world average value 254.6 ± 5.9 MeV for f_{D_s} given by the Heavy Flavor Averaging Group [22] based on the BaBar, Belle and CLEO measurements of $\mathcal{B}(D_s^+ \rightarrow \mu^+ \nu)$ and $\mathcal{B}(D_s^+ \rightarrow \tau^+ \nu)$. This value can be compared well to the results from the two precise lattice QCD calculations $f_{D_s} = 248.0 \pm 2.5$ MeV and 249 ± 11 MeV, respectively, from the HPQCD Collaboration [23] and the Fermilab/MILC Collaboration [24]. For the bottom sector, the Belle and BaBar collaborations have found evidence for $B^- \rightarrow \tau^- \nu$ decay in $e^+ e^- \rightarrow B^- B^+$

collisions at the $\Upsilon(4S)$ energy, however, the errors are rather large in the measured branching fractions with the computed average value $\mathcal{B}(B^- \rightarrow \tau^- \nu) = 1.72_{-0.42}^{+0.43} \times 10^{-4}$. Further a more accurate value of $|V_{ub}|$ is required for the determination of f_B . Considering the large uncertainties on V_{ub} and the branching fraction measurements for $B^- \rightarrow \tau^- \nu$, and sensitivity of this decay to the new physics, we rely upon $f_B = 193 \pm 11$ MeV, used in [21] as the average of the two lattice results $f_B = 195 \pm 11$ MeV [24] and $f_B = 190 \pm 13$ MeV [25], to fix the input parameter β_B . Likewise, for B_s meson, we use the lattice prediction of $f_{B_s} = 231 \pm 15$ MeV [25] for determining β_{B_s} .

(iii) The decay constants of the diagonal pseudoscalar mesons π^0, η, η' and η_c , in principle, could be obtained from $P \rightarrow \gamma\gamma$ branching fractions. In the case of π^0 , the value of $f_{\pi^0} = 130 \pm 5$ MeV [26] has been extracted from the measured $\pi^0 \rightarrow \gamma\gamma$ decay width, which is compatible with f_{π^\pm} , as is expected from isospin symmetry. However, decay constants of the $\eta - \eta'$ system cannot be extracted from two-photon decay rates alone and get more complicated due to the $\eta - \eta'$ mixing, the chiral anomaly and gluonium mixing [27, 28]. For describing the mixing between η and η' , it is more convenient to employ the flavor states $(u\bar{u} + d\bar{d})/\sqrt{2}$, and $(s\bar{s})$ labeled by the η_q and η_s , respectively. We then write

$$\begin{aligned}\eta &= \eta_q \cos \phi - \eta_s \sin \phi, \\ \eta' &= \eta_q \sin \phi + \eta_s \cos \phi,\end{aligned}\tag{3.16}$$

where $\phi = (39.3 \pm 1.0)^\circ$ follows from the analysis of Feldmann *et al.* [28] to fit the experimental data. This analysis also gives $f_\eta/f_\pi = 1.07 \pm 0.02$ and $f_{\eta'}/f_\pi = 1.34 \pm 0.06$, which are used in the present work. For η_c , the decay width is poorly known with PDG [20] estimate given as $\Gamma(\eta_c \rightarrow \gamma\gamma) = 7.2 \pm 2.1$ keV giving $f_{\eta_c} = 0.4 \pm 0.1$ GeV. Alternatively, one may extract f_{η_c} from $B \rightarrow \eta_c K$ decay using the factorization approximation, for which CLEO [29] obtained $f_{\eta_c} = 335 \pm 75$ MeV. In the literature, f_{η_c} is expected to be quite close to $f_{J/\psi}$ on the basis of quark model considerations [30]. Recently, the HPQCD collaboration [23] has reported a more precise result for f_{η_c} to be 394.7 ± 2.4 MeV consistent with other estimates, and is in fact very close to the experimental result $f_{J/\psi} = 410.6 \pm 6.2$ MeV obtained from the leptonic decay width of J/ψ [20]. So we use the lattice prediction to fix β_{η_c} . In the absence of any experimental estimate for f_{η_b} , we shall assume $f_{\eta_b} \approx f_\Upsilon$ to fix β_{η_b} following the heavy-quark spin symmetry.

(iv) For vector mesons, we extract the decay constants for diagonal states from the experimental values of their respective branching fractions of leptonic decays $V \rightarrow l^+ l^-$ decays [20]. Thus we obtain $f_{\rho^0} = 221.20 \pm 0.94$, $f_\omega = 194.60 \pm 3.24$, $f_\phi = 227.9 \pm 1.5$, $f_{J/\psi} = 410.6 \pm 6.2$ and $f_\Upsilon = 708.0 \pm 7.8$ (all in MeV) for ideal mixing, and use them to fix the β_V parameters of the respective mesons.

(v) The decay constant f_V determines not only the coupling of the neutral vector mesons to a photon, but also the coupling of charged vector mesons, like ρ^\pm and $K^{*\pm}$, to the weak vector bosons W^\pm . There are no data available for the leptonic decay of these charged vector mesons, but the couplings can be extracted indirectly from the decays $\tau \rightarrow \rho \nu_\tau$ and $\tau \rightarrow K^* \nu_\tau$. With the experimental values for the branching fractions of these decays $\mathcal{B}(\tau \rightarrow \rho \nu_\tau) = 25.02\%$ and

$\mathcal{B}(\tau \rightarrow K^* \nu_\tau) = 1.28\%$, the decay width formula

$$\Gamma(\tau \rightarrow V \nu_\tau) = \frac{G_F^2}{16\pi} |V_{q_1 q_2}|^2 f_V^2 \frac{(m_\tau^2 + 2m_V^2)(m_\tau^2 - m_V^2)^2}{m_\tau^3}, \quad (3.17)$$

where $V_{q_1 q_2}$ is the appropriate CKM- factor corresponding to the vector meson V , yield $f_{\rho^\pm} = 209 \pm 4$ MeV and $f_{K^{*\pm}} = 217 \pm 5$ MeV, respectively. It is worth noting that the difference in f_{ρ^0} and f_{ρ^\pm} seems consistent with the expected size of isospin breaking, and we take the average of the two values, i.e., $f_\rho = 215 \pm 5$ MeV, the error chosen so as to satisfy the two cases in extreme limits.

(vi) Contrary to the non-strange charmed meson case, where D^* has a slightly larger decay constant than D , the recent measurements of $B \rightarrow D_s^{(*)} D^{(*)}$ [20, 31] indicate that the decay constants of D_s^* and D_s are relatively similar. As for the decay constant of B^* , a recent lattice calculation yields $f_{B^*}/f_B = 1.01 \pm 0.01_{-0.01}^{+0.04}$ [32]. Explicitly, for naked charmed and bottom states D^*, D_s^*, B^* , and B_s^* , we have used the lattice predictions, $f_{D^*} = 245, f_{D_s^*} = 272, f_{B^*} = 196$, and $f_{B_s^*} = 229$ (all in MeV) [33] to fix the central value of the respective parameters β , and allow 10% variation in each case, giving decay constant ratios as $f_{D^*}/f_D = 1.18 \pm 0.17, f_{D_s^*}/f_{D_s} = 1.07 \pm 0.15$, and $f_{B^*}/f_B \approx f_{B_s^*}/f_{B_s} \approx 1.0 \pm 0.15$, to leave the scope for matching with other results.

(vii) For axial vector mesons, there are two different nonets of $J^P = 1^+$ in the quark model as the orbital excitation of the $q\bar{q}$ system. In terms of the spectroscopic notation $^{2S+1}L_J$, there are two types of p -wave axial vector mesons, namely, 3P_1 and 1P_1 , which have distinctive C quantum numbers, $C = +$ and $C = -$, respectively. Experimentally, the $J^{PC} = 1^{++}$ nonet consists of $a_1(1260), f_1(1285), f_1(1420)$, and K_{1A} , while the $J^{PC} = 1^{+-}$ nonet has $b_1(1235), h_1(1170), h_1(1380)$, and K_{1B} .

(viii) It is generally argued that $a_1(1260)$ should have a similar decay constant as the ρ meson. Presumably, f_{a_1} can be extracted from the decay $\tau \rightarrow a_1(1260)\nu_\tau$. Though this decay is not shown in the Particle Data Group [20], an experimental value of $|f_{a_1}| = 203 \pm 18$ MeV is nevertheless quoted in [34].² The $a_1(1260)$ decay constant $f_{a_1} = 238 \pm 10$ MeV obtained using the QCD sum rule method [35] is slightly higher than this value as well as $f_\rho = 215$ MeV. In Table III we have employed $f_{a_1} = -203 \pm 18$ MeV as input following our sign convention.

(ix) The nonstrange axial-vector mesons, for example, $a_1(1260)$ and $b_1(1235)$ cannot have mixing because of the opposite C -parities. On the contrary, physical strange axial-vector mesons are the mixture of 3P_1 and 1P_1 states, while the heavy axial-vector resonances are generally taken as the mixture of $P_1^{1/2}$ and $P_1^{3/2}$. For example, the physical mass eigenstates $K_1(1270)$ and $K_1(1400)$ are a mixture of K_{1A} and K_{1B} states owing to the mass difference of the strange and nonstrange light quarks:

$$\begin{aligned} K_1(1270) &= K_{1A} \sin \theta_{K_1} + K_{1B} \cos \theta_{K_1}, \\ K_1(1400) &= K_{1A} \cos \theta_{K_1} - K_{1B} \sin \theta_{K_1}. \end{aligned} \quad (3.18)$$

Using the experimental results $\mathcal{B}(\tau \rightarrow K_1(1270)\nu_\tau) = (4.7 \pm 1.1) \times 10^{-3}$ and $\Gamma(\tau \rightarrow K_1(1270)\nu_\tau)/[\Gamma(\tau \rightarrow K_1(1270)\nu_\tau) + \Gamma(\tau \rightarrow K_1(1400)\nu_\tau)] = 0.69 \pm 0.15$ [20], and the decay width

² The decay constant of a_1 can be tested in the decay $B^+ \rightarrow \bar{D}^0 a_1^+$ which receives the main contribution from the color-allowed amplitude proportional to $f_{a_1} F^{BD}(m_{a_1}^2)$.

formula similar to that given in Eq. (3.17) with the replacement $V \rightarrow A$, we obtain ³

$$\begin{aligned} |f_{K_1(1270)}| &= 169.5^{+18.8}_{-21.2} \text{ MeV}, \\ |f_{K_1(1400)}| &= 139.2^{+41.3}_{-45.6} \text{ MeV}. \end{aligned} \quad (3.19)$$

These decay constants are related to $f_{K_{1A}}$ and $f_{K_{1B}}$ through

$$\begin{aligned} m_{K_1(1270)} f_{K_1(1270)} &= m_{K_{1A}} f_{K_{1A}} \sin \theta_{K_1} + m_{K_{1B}} f_{K_{1B}} \cos \theta_{K_1}, \\ m_{K_1(1400)} f_{K_1(1400)} &= m_{K_{1A}} f_{K_{1A}} \cos \theta_{K_1} - m_{K_{1B}} f_{K_{1B}} \sin \theta_{K_1}, \end{aligned} \quad (3.20)$$

where use of Eq. (3.18) and expressions for decay constants have been made. From the analytic expressions of decay constants given in Eq. (3.1), it is clear that $m_{K_{1A}} f_{K_{1A}}$ and $m_{K_{1B}} f_{K_{1B}}$ are functions of β_{K_1} and quark masses only. In other words, they do not depend on $m_{K_{1A,1B}}$ and hence θ_{K_1} . Eq. (3.20) leads to the following relation:

$$m_{K_1(1270)}^2 f_{K_1(1270)}^2 + m_{K_1(1400)}^2 f_{K_1(1400)}^2 = m_{K_{1A}}^2 f_{K_{1A}}^2 + m_{K_{1B}}^2 f_{K_{1B}}^2. \quad (3.21)$$

This relation, being independent of the mixing angle θ_{K_1} , has been used [7] to determine the central value of the parameter β_{K_1} to be 0.3224 GeV. However, to calculate the individual decay constants, masses of K_1 mesons are needed for which the mixing angle θ_{K_1} is required. From Eq. (3.18), the masses of the K_{1A} and K_{1B} can be expressed as

$$\begin{aligned} m_{K_{1A}}^2 &= m_{K_1(1270)}^2 \sin^2 \theta_{K_1} + m_{K_1(1400)}^2 \cos^2 \theta_{K_1}, \\ m_{K_{1B}}^2 &= m_{K_1(1270)}^2 \cos^2 \theta_{K_1} + m_{K_1(1400)}^2 \sin^2 \theta_{K_1}. \end{aligned} \quad (3.22)$$

There exists several estimations on the mixing angle in the literature [14, 36, 37] differing in the value and sign convention. These often employ masses, partial decay rates of $K_1(1270)$ and $K_1(1400)$, and τ decay rates to these mesons.⁴ Note that in the CLF quark model, the sign of $f_{K_{1A}}$ is negative, whereas $f_{K_{1B}}$ is positive. With this sign convention, from Eq. (3.20), the following two solutions [7] have been obtained:

$$\theta_{K_1} = \begin{cases} +50.8^\circ \text{ solution I,} \\ -44.8^\circ \text{ solution II.} \end{cases} \quad (3.23)$$

The second solution is ruled out by the experimental data for $B \rightarrow K_1(1270)/K_1(1400) + \gamma$ decays [7]. For $\theta_{K_1} = 50.8^\circ$, masses of 3P_1 and 1P_1 states come out to be,

$$m_{K_{1A}} = 1.26 \text{ GeV}, \quad m_{K_{1B}} = 1.52 \text{ GeV}, \quad (3.24)$$

corresponding to

$$f_{K_1(1270)} = -170 \text{ MeV}, \quad f_{K_1(1400)} = 139 \text{ MeV}. \quad (3.25)$$

³ The large experimental error with the $K_1(1400)$ production in the τ decays, namely $\mathcal{B}(\tau \rightarrow K_1(1400)\nu_\tau) = (1.7 \pm 2.6) \times 10^{-3}$ [20], does not provide sensible information for the $K_1(1400)$ decay constant

⁴ The relative signs of the decay constants, form factors, and mixing angles of the axial vector mesons were often confusing in the literature. The sign of mixing angle is intimately related to the relative sign of the K_{1A} and K_{1B} states. For a detailed discussion, refer to [7, 38, 39].

Note that obtained value for β_{K_1} lies between β_K and β_{K^*} .

(x) Like s -wave mesons, there are mixing between singlet and octet states of p -wave mesons also, equivalently, between $u\bar{u} + d\bar{d}$ and $s\bar{s}$ components. For axial vector states $f_1(1285)$ and $f_1(1420)$, the mixing can be written as

$$\begin{aligned} f_1(1285) &= f_{1q} \sin \alpha_{f_1} + f_{1s} \cos \alpha_{f_1}, \\ f_1(1420) &= f_{1q} \cos \alpha_{f_1} - f_{1s} \sin \alpha_{f_1}, \end{aligned} \quad (3.26)$$

where $f_{1q} = (u\bar{u} + d\bar{d})/\sqrt{2}$, and f_{1s} is pure $(s\bar{s})$ state. The mixing angle α_{f_1} is related to the singlet-octet mixing angle θ_{f_1} by the $\alpha_{f_1} = \theta_{f_1} + 54.7^\circ$, where the latter mixing angle is defined by

$$\begin{aligned} f_1(1285) &= f_1 \cos \theta_{f_1} + f_8 \sin \theta_{f_1}, \\ f_1(1420) &= -f_1 \sin \theta_{f_1} + f_8 \cos \theta_{f_1}. \end{aligned} \quad (3.27)$$

The magnitude of the angle is given by the mass relations

$$\tan^2 \theta_{f_1} = \frac{4m_{K_{1A}}^2 - m_{a_1}^2 - 3m_{f_1(1420)}^2}{-4m_{K_{1A}}^2 + m_{a_1}^2 + 3m_{f_1(1285)}^2}, \quad (3.28)$$

while the sign of the angle can be determined from

$$\tan \theta_{f_1} = \frac{4m_{K_{1A}}^2 - m_{a_1}^2 - 3m_{f_1(1420)}^2}{2\sqrt{2}(m_{a_1}^2 - m_{K_{1A}}^2)}. \quad (3.29)$$

We thus obtain $\alpha_{f_1} = 94.9^\circ$, i.e., $\theta_{f_1} = 40.2^\circ$. Denoting the mass of the f_{1s} component as $m_{f_{1s}}$, we have

$$m_{f_{1s}}^2 = m_{f_1(1420)}^2 \sin^2 \alpha_{f_1} + m_{f_1(1285)}^2 \cos^2 \alpha_{f_1}, \quad (3.30)$$

which yields $m_{f_{1s}} = 1.425$ GeV. Using the mixing angle and $m_{s\bar{s}}$, the decay constant $f_{f_{1s}}$ of the 3P_1 axial vector meson with a pure $s\bar{s}$ quark content has been determined to be -230 ± 9 MeV [40]. Consequently, $\beta_{f_{1s}}$ gets fixed in the present CLF model to be 0.3492 ± 0.0064 [7]. For the purpose of an estimation, for the remaining axial vector mesons, we use the β parameters obtained in the ISGW2 model [11] up to some simple scaling, and 10% uncertainty has been assigned to them arbitrarily to study its effects on their decay constants and the corresponding form factors.

(xi) The β values are kept same for other p -wave mesons, scalar ($J^{PC} = 0^{++}$) and axial vector ($J^{PC} = 1^{+-}$) mesons, as that of the ($J^{PC} = 1^{++}$) mesons having the same flavor quantum numbers. So their decay constants are calculated respectively as shown in Table II. The β parameters for p -wave states of the charmed and bottom states are smaller when compared to the respective $\beta_{P,V}$ values.

(xii) Situation regarding the decay constant for the 1P_1 mesons is different from the 3P_1 mesons. First of all, its decay constant vanishes in the isospin or SU(3) limit. In fact, because of charge conjugation invariance, the decay constant of the nonstrange neutral meson $b_1^0(1235)$ must be zero. In the isospin limit, the decay constant of the charged b_1 vanishes due to the fact that the b_1 has even G -parity and that the relevant weak axial-vector current is odd under G transformation. Hence, $f_{b_1^+(1235)}$ is very small in reality, arising due to the small mass difference between u and d

quark masses. In the present covariant light-front quark model, if we increase the constituent d quark mass by an amount of 5 ± 2 MeV relative to the u quark one, we find $f_{b_1^+(1235)} = 0.6 \pm 0.2$ MeV which is highly suppressed.⁵ Similar to the $f_1(1285) - f_1(1420)$ mixing in the $J^{PC} = 1^{++}$ nonet, the $h_1(1170) - h_1(1380)$ mixing can be described for $J^{PC} = 1^{+-}$ with the replacement $f_1(1285) \rightarrow h_1(1170)$, $f_1(1420) \rightarrow h_1(1380)$, and $\theta_{f_1} \rightarrow \theta_{h_1}$, $\alpha_{f_1} \rightarrow \alpha_{h_1}$, leading to $\alpha_{h_1} = 54.7^\circ$ [7], i.e., $\theta_{h_1} = 0^\circ$. However, like b_1^0 , decay constants of these mesons also vanish. In fact, in the SU(3) limit, $f_{1P_1} = 0$ should follow for strange mesons also in this nonet. However, for the strange mesons K_{1B} , nonzero decay constant would arise through SU(3) breaking, and we obtain $f_{K_{1B}} = 20.4^{+2.5}_{-1.8}$ MeV. For charmed and bottom axial vector mesons, the results given in Table III translate to $f_{D_1^{1/2}} = 179^{+37}_{-34}$, $f_{D_1^{3/2}} = -53.6^{+14.0}_{-12.1}$, $f_{D_{1s}^{1/2}} = 154^{+34}_{-31}$, $f_{D_{1s}^{3/2}} = -57.3^{+14.7}_{-12.8}$, $f_{B_1^{1/2}} = 175^{+33}_{-30}$, $f_{B_1^{3/2}} = -21.4^{+5.7}_{-4.9}$, $f_{B_{1s}^{1/2}} = 183^{+37}_{-34}$, $f_{B_{1s}^{3/2}} = -28.3^{+7.5}_{-6.4}$, $f_{B_{1c}^{1/2}} = 157^{+37}_{-33}$, and $f_{B_{1c}^{3/2}} = -47.3^{+12.4}_{-10.7}$ (all in MeV). The errors shown here occur due to the 10% arbitrary uncertainty assigned to their β values. Note that the decay constants of 3P_1 and $P_1^{3/2}$ states have opposite signs to that of 1P_1 or $P_1^{1/2}$ as can be easily seen from Eq. (3.4).

(xiii) Similarly for scalar mesons, their decay constants also vanish in the SU(N) limit, as has been shown above Eq.(3.2) by applying equations of motion. However, due to SU(N) breaking, only off-diagonal scalar mesons can have nonzero decay constants, which have been given in Table III. In the present covariant light-front quark model, if the constituent d quark mass is increased by an amount of 5 ± 2 MeV relative to the u quark one, we find $|f_{a_0^\pm(1450)}| = 1.1 \pm 0.4$ MeV which is highly suppressed, whereas SU(3) breaking yields $|f_{K_0^{*\pm}}| \approx 35$ MeV. Thus it is clear that the decay constant of light scalar resonances remain largely suppressed relative to that of the pseudoscalar mesons owing to the small mass difference between the constituent quark masses, though this suppression becomes less restrictive for heavy scalar mesons because of heavy and light quark mass imbalance, and decay constants are of the order of hundred MeV. Note that what is the underlying quark structure of light scalar resonances is still controversial. While it has been widely advocated that the light scalar nonet formed by $\sigma(600)$, $\kappa(800)$, $f_0(980)$ and $a_0(980)$ can be identified primarily as four-quark states, it is generally believed that the nonet states $a_0(1450)$, $K_0^*(1430)$, $f_0(1370)$ and $f_0(1500)/f_0(1710)$ are the conventional $q\bar{q}'$ states [42]. Therefore, the prediction of $f_{K_0^*} \approx 35$ MeV for the scalar meson in the $s\bar{u}$ content (see Table III) is most likely designated for the $K_0^*(1430)$ state. Notice that this prediction is slightly smaller than the result of 42 MeV obtained in [43] based on the finite-energy sum rules, and far less than the estimate of (70 ± 10) MeV in [44]. It is worth remarking that even if the light scalar mesons are made from 4 quarks, the decay constants of the neutral scalars $\sigma(600)$, $f_0(980)$ and $a_0^0(980)$ must vanish owing to charge conjugation invariance.

(xiv) In this work, we have only considered the scalar nonet with masses above 1 GeV, for which the quark content of $a_0(1450)$ and $K_0^*(1430)$ is quite obvious, whereas the internal structure of the isoscalars $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ in the same nonet is controversial and less clear. Since

⁵ In [41], the decay constants of a_1 and b_1 are derived using the $K_{1A} - K_{1B}$ mixing angle θ_{K_1} and SU(3) symmetry to be $(f_{b_1}; f_{a_1}) = (74; 215)$ MeV for $\theta_{K_1} = 32^\circ$ and $(-28; 223)$ MeV for $\theta_{K_1} = 58^\circ$. It seems to us that the magnitude of b_1 decay constant derived in this manner is too big.

not all the three isosinglet scalars can be accommodated in the $q\bar{q}$ nonet picture, one of them should be primarily a scalar glueball. Among them, it has been quite controversial as to which of these is the dominant scalar glueball. It has been advocated that $f_0(1710)$ is mainly $(s\bar{s})$ and $f_0(1500)$ mostly gluonic [45, 46]. However, this scenario encounters several insurmountable difficulties, see [47, 48] for detailed discussions. Based on two simple and robust results as the input for the mass matrix, the analysis in [48] shows that in the limit of exact SU(3) symmetry, $f_0(1500)$ is an SU(3) isosinglet octet state and is degenerate with $a_0(1450)$. In the absence of glueball-quarkonium mixing, $f_0(1370)$ becomes a pure SU(3) singlet and $f_0(1710)$ the pure glueball. When the glueball-quarkonium mixing is turned on, there will be some mixing between the glueball (G) and the SU(3)-singlet $q\bar{q}$. The mixing matrix obtained in this model has the form [48]:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.78 & 0.51 & -0.36 \\ -0.54 & 0.84 & 0.03 \\ 0.32 & 0.18 & 0.93 \end{pmatrix} \begin{pmatrix} f_{0q} \\ f_{0s} \\ G \end{pmatrix}, \quad (3.31)$$

where $f_{0q} = (u\bar{u} + d\bar{d})/\sqrt{2}$, and f_{0s} is pure $(s\bar{s})$ state, with masses 1.474 GeV and 1.5 GeV, respectively. It is evident that $f_0(1710)$ is composed primarily of the scalar glueball, $f_0(1500)$ is close to an SU(3) octet, and $f_0(1370)$ consists of an approximate SU(3) singlet with some glueball component ($\sim 10\%$). Note that the recent quenched and unquenched lattice calculations all favor a scalar glueball mass close to 1700 MeV [49].

(xv) In principle, the decay constant of the scalar strange charmed meson D_{s0}^* can be determined from the hadronic decay $B \rightarrow \bar{D}D_{s0}^*$ since it proceeds only via external W -emission. Indeed, a measurement of the $D\bar{D}_{s0}^*$ production in B decays by Belle [50] indicates a $f_{D_{s0}^*}$ of order 60 MeV [51] which is close to the calculated value of 71 MeV (see Table III). In our earlier work [1], we have discussed more about $\bar{D}D_s^{**}$ productions in B decays. The smallness of the decay constant $f_{D_{s0}^*}$ relative to f_{D_s} can be seen from Eqs. (3.6) and (3.8) that

$$f_{D_s(D_{s0}^*)} \propto \int dx_2 \cdots [m_c x_2 \pm m_s(1 - x_2)]. \quad (3.32)$$

Since the momentum fraction x_2 of the strange quark in the $D_s(D_{s0}^*)$ meson is small, its effect being constructive in D_s case and destructive in D_{s0}^* is sizable and explains why $f_{D_{s0}^*}/f_{D_s} \sim 0.2$.

(xvi) For D and B systems, it is clear from Table III that $|f_{A^{3/2}}| \ll f_S < f_{A^{1/2}}$, in accordance with the expectation from HQS [cf. Eq. (3.5)]. Decay constants of p -wave charmed and bottom mesons have been obtained using the Bethe-Salpeter method [52], which are consistent with values obtained in Table II for the bottom sector and 1P_1 charmed mesons, and are slightly higher than that of other p -wave charmed mesons. However, our values for D_{s0}^* and D_{s1} match well with the results $f_{D_{s0}^*} = 67.1 \pm 4.5$ MeV and $f_{D_{s1}} = 144.5 \pm 11.1$ MeV obtained in [53] based on the analysis of $B \rightarrow D^* + D_{s0}^*/D_{s1}$ decays. For charmed ($c\bar{u}$) meson, our estimate $f_{D_1^{1/2}} = 179_{-34}^{+37}$ MeV is consistent with values $f_{D_1^{1/2}} = 196 \pm 93$ MeV and 206 ± 120 MeV obtained in [54] from the analysis of $\bar{B} \rightarrow D_1^{1/2}\pi$ and $\bar{B} \rightarrow D_0^{1/2}\pi$ decays, respectively.

(xvii) The β values used in the present analysis often differ from the ones given in the earlier work [1] to match with the decay constants based on the latest data. For the same reason, the strange quark mass $m_s = 0.45$ GeV used here is different from the values 0.37 GeV used earlier [1]. This

choice of the strange quark mass has to be made to obtain $f_{K_0^*} = 0.35$ MeV based on the analysis of the B decays emitting K_0^* [55, 56]. Otherwise, for the strange quark mass $m_s = 0.37$ GeV, this decay constant would require $\beta_{K_0^*} > 0.60$, which is quite high for p -wave mesons. Particularly, it would also spoil the matching for decay constants of axial vector K_1 mesons which seem to require $\beta < 0.4$. Furthermore, choosing $\beta_{K_0^*} > 0.60$, would also enhance $f_{D_{s0}^*}$ and $f_{D_{s1}^{1/2}}$ unduly high if their β 's are taken to be greater than or equal to that of K_0^* . The new choice of $m_s = 0.45$ GeV has obviously resulted in difference in the obtained form factors involving strange mesons from that given in the earlier work [1].

(xviii) In this work, we have investigated the variation of the form factors and their slope parameters for q^2 dependence with the variation of β values. Wherever the experimental information is available, we have used that to fix the errors in the beta values, otherwise a standard 10% uncertainty in β is assigned to the remaining s -wave and p -wave mesons.

IV. COVARIANT MODEL ANALYSIS OF FORM FACTORS

In this section we first describe the form factors for s -wave mesons within the framework of the covariant light-front quark model [4] and then extend it to the p -wave meson case followed by numerical results and discussions in the next section.

A. Form factors for s -wave to s -wave transitions

Form factors for $P \rightarrow P, V$ transitions are defined by

$$\begin{aligned}\langle P(P'')|V_\mu|P(P')\rangle &= P_\mu f_+(q^2) + q_\mu f_-(q^2), \\ \langle V(P'', \varepsilon'')|V_\mu|P(P')\rangle &= \epsilon_{\mu\nu\alpha\beta} \varepsilon''^{*\nu} P^\alpha q^\beta g(q^2), \\ \langle V(P'', \varepsilon'')|A_\mu|P(P')\rangle &= -i \left\{ \varepsilon_\mu''^* f(q^2) + \varepsilon''^{*} \cdot P \left[P_\mu a_+(q^2) + q_\mu a_-(q^2) \right] \right\},\end{aligned}\quad (4.1)$$

where $P = P' + P''$, $q = P' - P''$ and the convention $\epsilon_{0123} = 1$ is adopted. These form factors are related to the commonly used Bauer-Stech-Wirbel (BSW) form factors [57] via

$$\begin{aligned}F_1^{PP}(q^2) &= f_+(q^2), \quad F_0^{PP}(q^2) = f_+(q^2) + \frac{q^2}{q \cdot P} f_-(q^2), \\ V^{PV}(q^2) &= -(M' + M'') g(q^2), \quad A_1^{PV}(q^2) = -\frac{f(q^2)}{M' + M''}, \\ A_2^{PV}(q^2) &= (M' + M'') a_+(q^2), \quad A_3^{PV}(q^2) - A_0^{PV}(q^2) = \frac{q^2}{2M''} a_-(q^2),\end{aligned}\quad (4.2)$$

where the latter form factors are defined by [57]

$$\begin{aligned}\langle P(P'')|V_\mu|P(P')\rangle &= \left(P_\mu - \frac{M'^2 - M''^2}{q^2} q_\mu \right) F_1^{PP}(q^2) + \frac{M'^2 - M''^2}{q^2} q_\mu F_0^{PP}(q^2), \\ \langle V(P'', \varepsilon'')|V_\mu|P(P')\rangle &= -\frac{1}{M' + M''} \epsilon_{\mu\nu\alpha\beta} \varepsilon''^{*\nu} P^\alpha q^\beta V^{PV}(q^2), \\ \langle V(P'', \varepsilon'')|A_\mu|P(P')\rangle &= i \left\{ (M' + M'') \varepsilon_\mu''^* A_1^{PV}(q^2) - \frac{\varepsilon''^{*} \cdot P}{M' + M''} P_\mu A_2^{PV}(q^2) \right\}\end{aligned}$$

$$-2M'' \frac{\varepsilon''^* \cdot P}{q^2} q_\mu [A_3^{PV}(q^2) - A_0^{PV}(q^2)] \}, \quad (4.3)$$

with $F_1^{PP}(0) = F_0^{PP}(0)$, $A_3^{PV}(0) = A_0^{PV}(0)$, and

$$A_3^{PV}(q^2) = \frac{M' + M''}{2M''} A_1^{PV}(q^2) - \frac{M' - M''}{2M''} A_2^{PV}(q^2). \quad (4.4)$$

Besides the dimensionless form factors, this parametrization has the advantage that the q^2 dependence of the form factors is governed by the resonances of the same spin, for instance, the momentum dependence of $F_0(q^2)$ is determined by scalar resonances.

To obtain the $P \rightarrow M$ transition form factors with M being a ground-state s -wave meson (or a low-lying p -wave meson), we follow [1, 4] to obtain $P \rightarrow P, V$ form factors before considering the p -wave meson case. For the case of $M = P$, it is straightforward to obtain the form factors $f_\pm(q^2)$ for $q^2 = -q_\perp^2 \leq 0$. We will return to the issue of the momentum dependence of form factors in the last sub-section. At $q^2 = 0$, the form factor $f_+(0)$ is simply given by

$$f_+(q^2) = \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_P h''_P}{x_2 \hat{N}'_1 \hat{N}''_1} \left[x_1 (M_0'^2 + M_0''^2) + x_2 q^2 \right. \\ \left. - x_2 (m'_1 - m''_1)^2 - x_1 (m'_1 - m_2)^2 - x_1 (m''_1 - m_2)^2 \right]. \quad (4.5)$$

Similarly, we have

$$f_-(q^2) = \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{2h'_P h''_P}{x_2 \hat{N}'_1 \hat{N}''_1} \left\{ -x_1 x_2 M'^2 - p_\perp'^2 - m'_1 m_2 + (m''_1 - m_2)(x_2 m'_1 + x_1 m_2) \right. \\ \left. + 2 \frac{q \cdot P}{q^2} \left(p_\perp'^2 + 2 \frac{(p'_\perp \cdot q_\perp)^2}{q^2} \right) + 2 \frac{(p'_\perp \cdot q_\perp)^2}{q^2} - \frac{p'_\perp \cdot q_\perp}{q^2} [M''^2 - x_2 (q^2 + q \cdot P) \right. \\ \left. - (x_2 - x_1) M'^2 + 2x_1 M_0'^2 - 2(m'_1 - m_2)(m'_1 + m''_1)] \right\}. \quad (4.6)$$

We next turn to the $P \rightarrow V$ transition form factors, which are given by

$$g(q^2) = -\frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{2h'_P h''_V}{x_2 \hat{N}'_1 \hat{N}''_1} \left\{ x_2 m'_1 + x_1 m_2 + (m'_1 - m''_1) \frac{p'_\perp \cdot q_\perp}{q^2} + \frac{2}{w_V''} \left[p_\perp'^2 + \frac{(p'_\perp \cdot q_\perp)^2}{q^2} \right] \right\}, \\ f(q^2) = \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_P h''_V}{x_2 \hat{N}'_1 \hat{N}''_1} \left\{ 2x_1 (m_2 - m'_1)(M_0'^2 + M_0''^2) - 4x_1 m''_1 M_0'^2 + 2x_2 m'_1 q \cdot P \right. \\ \left. + 2m_2 q^2 - 2x_1 m_2 (M'^2 + M''^2) + 2(m'_1 - m_2)(m'_1 + m''_1)^2 + 8(m'_1 - m_2) \left[p_\perp'^2 + \frac{(p'_\perp \cdot q_\perp)^2}{q^2} \right] \right. \\ \left. + 2(m'_1 + m''_1)(q^2 + q \cdot P) \frac{p'_\perp \cdot q_\perp}{q^2} - 4 \frac{q^2 p_\perp'^2 + (p'_\perp \cdot q_\perp)^2}{q^2 w_V''} \left[2x_1 (M'^2 + M_0'^2) - q^2 - q \cdot P \right. \right. \\ \left. \left. - 2(q^2 + q \cdot P) \frac{p'_\perp \cdot q_\perp}{q^2} - 2(m'_1 - m''_1)(m'_1 - m_2) \right] \right\}, \\ a_+(q^2) = \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{2h'_P h''_V}{x_2 \hat{N}'_1 \hat{N}''_1} \left\{ (x_1 - x_2)(x_2 m'_1 + x_1 m_2) - [2x_1 m_2 + m''_1 + (x_2 - x_1)m'_1] \frac{p'_\perp \cdot q_\perp}{q^2} \right\}$$

$$\begin{aligned}
& -2 \frac{x_2 q^2 + p'_\perp \cdot q_\perp}{x_2 q^2 w_V''} \left[p'_\perp \cdot p''_\perp + (x_1 m_2 + x_2 m'_1)(x_1 m_2 - x_2 m''_1) \right] \Big\}, \\
a_-(q^2) = & \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_P h''_V}{x_2 \hat{N}'_1 \hat{N}''_1} \left\{ 2(2x_1 - 3)(x_2 m'_1 + x_1 m_2) - 8(m'_1 - m_2) \left[\frac{p'^2_\perp}{q^2} + 2 \frac{(p'_\perp \cdot q_\perp)^2}{q^4} \right] \right. \\
& - [(14 - 12x_1)m'_1 - 2m''_1 - (8 - 12x_1)m_2] \frac{p'_\perp \cdot q_\perp}{q^2} \\
& + \frac{4}{w_V''} \left([M'^2 + M''^2 - q^2 + 2(m'_1 - m_2)(m''_1 + m_2)](A_3^{(2)} + A_4^{(2)} - A_2^{(1)}) \right. \\
& + Z_2(3A_2^{(1)} - 2A_4^{(2)} - 1) + \frac{1}{2}[x_1(q^2 + q \cdot P) - 2M'^2 - 2p'_\perp \cdot q_\perp \\
& - 2m'_1(m''_1 + m_2) - 2m_2(m'_1 - m_2)](A_1^{(1)} + A_2^{(1)} - 1) \\
& \left. \left. + q \cdot P \left[\frac{p'^2_\perp}{q^2} + \frac{(p'_\perp \cdot q_\perp)^2}{q^4} \right] (4A_2^{(1)} - 3) \right] \right\}, \tag{4.7}
\end{aligned}$$

where various quantities appearing in these formulas have been described in the previous section.

B. Form factors for s -wave to p -wave transitions

The general expressions for P to low-lying p -wave meson transitions are given by [9]

$$\begin{aligned}
\langle S(P'') | A_\mu | P(P') \rangle &= i \left[u_+(q^2) P_\mu + u_-(q^2) q_\mu \right], \\
\langle A^{1/2}(P'', \varepsilon'') | V_\mu | P(P') \rangle &= i \left\{ \ell_{1/2}(q^2) \varepsilon''^*_{\mu} + \varepsilon''^* \cdot P [P_\mu c_+^{1/2}(q^2) + q_\mu c_-^{1/2}(q^2)] \right\}, \\
\langle A^{1/2}(P'', \varepsilon'') | A_\mu | P(P') \rangle &= -q_{1/2}(q^2) \epsilon_{\mu\nu\alpha\beta} \varepsilon''^{*\nu} P^\alpha q^\beta, \\
\langle A^{3/2}(P'', \varepsilon'') | V_\mu | P(P') \rangle &= i \left\{ \ell_{3/2}(q^2) \varepsilon''^*_{\mu} + \varepsilon''^* \cdot P [P_\mu c_+^{3/2}(q^2) + q_\mu c_-^{3/2}(q^2)] \right\}, \\
\langle A^{3/2}(P'', \varepsilon'') | A_\mu | P(P') \rangle &= -q_{3/2}(q^2) \epsilon_{\mu\nu\alpha\beta} \varepsilon''^{*\nu} P^\alpha q^\beta. \tag{4.8}
\end{aligned}$$

The form factors $\ell_{1/2(3/2)}$, $c_+^{1/2(3/2)}$, $c_-^{1/2(3/2)}$ and $q_{1/2(3/2)}$ are defined for the transitions to the heavy $P_1^{1/2}$ ($P_1^{3/2}$) state. For transitions to light axial-vector mesons, it is more appropriate to employ the $L-S$ coupled states 1P_1 and 3P_1 denoted by the particles 1A and 3A in our notation. The relation between $P_1^{1/2}$, $P_1^{3/2}$ and 1P_1 , 3P_1 states is given by Eq. (3.4). The corresponding form factors $\ell_{1A(^3A)}$, $c_+^{1A(^3A)}$, $c_-^{1A(^3A)}$ and $q_{1A(^3A)}$ for $P \rightarrow ^1A$ (3A) transitions can be defined in an analogous way.⁶

Note that only the form factors $u_+(q^2)$, $u_-(q^2)$ and $k(q^2)$ in the above parametrization are dimensionless. It is thus convenient to define dimensionless form factors by⁷

$$\langle S(P'') | A_\mu | P(P') \rangle = -i \left[\left(P_\mu - \frac{M'^2 - M''^2}{q^2} q_\mu \right) F_1^{PS}(q^2) + \frac{M'^2 - M''^2}{q^2} q_\mu F_0^{PS}(q^2) \right],$$

⁶ The form factors $\ell_{1A(^3A)}$, $c_+^{1A(^3A)}$, $c_-^{1A(^3A)}$ and $q_{1A(^3A)}$ are dubbed as $\ell(v)$, $c_+(s_+)$, $c_-(s_-)$ and $q(r)$, respectively, in the ISGW model [9].

⁷ The definition here for dimensionless $P \rightarrow A$ transition form factors differs than Eq. (3.17) of [51] where the coefficients $(m_P \pm m_A)$ are replaced by $(m_P \mp m_A)$. It has been made clear in the earlier work [1] that this definition will lead to HQS relations for $B \rightarrow D_0^*$, D_1 transitions similar to that for $B \rightarrow D$, D^* ones.

$$\begin{aligned}
\langle A(P'', \varepsilon'') | V_\mu | P(P') \rangle &= -i \left\{ (m_P - m_A) \varepsilon_\mu^* V_1^{PA}(q^2) - \frac{\varepsilon^* \cdot P'}{m_P - m_A} P_\mu V_2^{PA}(q^2) \right. \\
&\quad \left. - 2m_A \frac{\varepsilon^* \cdot P'}{q^2} q_\mu [V_3^{PA}(q^2) - V_0^{PA}(q^2)] \right\}, \\
\langle A(P'', \varepsilon'') | A_\mu | P(P') \rangle &= -\frac{1}{m_P - m_A} \epsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} P^\rho q^\sigma A^{PA}(q^2),
\end{aligned} \tag{4.9}$$

with

$$V_3^{PA}(q^2) = \frac{m_P - m_A}{2m_A} V_1^{PA}(q^2) - \frac{m_P + m_A}{2m_A} V_2^{PA}(q^2), \tag{4.10}$$

and $V_3^{PA}(0) = V_0^{PA}(0)$. They are related to the form factors in (4.3) via

$$\begin{aligned}
F_1^{PS}(q^2) &= -u_+(q^2), \quad F_0^{PS}(q^2) = -u_+(q^2) - \frac{q^2}{q \cdot P} u_-(q^2), \\
A^{PA}(q^2) &= -(M' - M'') q(q^2), \quad V_1^{PA}(q^2) = -\frac{\ell(q^2)}{M' - M''}, \\
V_2^{PA}(q^2) &= (M' - M'') c_+(q^2), \quad V_3^{PA}(q^2) - V_0^{PA}(q^2) = \frac{q^2}{2M''} c_-(q^2).
\end{aligned} \tag{4.11}$$

In above equations, the axial-vector meson A stands for $A^{1/2}$ or $A^{3/2}$.

The $P \rightarrow S(A)$ transition form factors can be easily obtained by some suitable modifications on $P \rightarrow P(V)$ ones. The $P \rightarrow S$ transition form factors are related to f_\pm by

$$u_\pm = -f_\pm(m_1'' \rightarrow -m_1'', h_P'' \rightarrow h_S''). \tag{4.12}$$

Thus the following form of these form factors can be obtained from that of $P \rightarrow P$ ones by the replacements given above,

$$\begin{aligned}
u_+(q^2) &= \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_P h''_S}{x_2 \hat{N}'_1 \hat{N}''_1} \left[-x_1 (M_0'^2 + M_0''^2) - x_2 q^2 \right. \\
&\quad \left. + x_2 (m'_1 + m''_1)^2 + x_1 (m'_1 - m_2)^2 + x_1 (m''_1 + m_2)^2 \right], \\
u_-(q^2) &= \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{2h'_P h''_S}{x_2 \hat{N}'_1 \hat{N}''_1} \left\{ x_1 x_2 M'^2 + p_\perp'^2 + m'_1 m_2 + (m''_1 + m_2)(x_2 m'_1 + x_1 m_2) \right. \\
&\quad \left. - 2 \frac{q \cdot P}{q^2} \left(p_\perp'^2 + 2 \frac{(p'_\perp \cdot q_\perp)^2}{q^2} \right) - 2 \frac{(p'_\perp \cdot q_\perp)^2}{q^2} + \frac{p'_\perp \cdot q_\perp}{q^2} [M''^2 - x_2 (q^2 + q \cdot P) \right. \\
&\quad \left. \left. - (x_2 - x_1) M'^2 + 2x_1 M_0'^2 - 2(m'_1 - m_2)(m'_1 - m''_1) \right] \right\}.
\end{aligned} \tag{4.13}$$

Similarly, the analytic expressions for $P \rightarrow A$ transition form factors can be obtained from that of $P \rightarrow V$ ones by the following replacements:

$$\begin{aligned}
\ell^{3A,1A}(q^2) &= f(q^2) \text{ with } (m_1'' \rightarrow -m_1'', h_V'' \rightarrow h_{3A,1A}'', w_V'' \rightarrow w_{3A,1A}''), \\
q^{3A,1A}(q^2) &= g(q^2) \text{ with } (m_1'' \rightarrow -m_1'', h_V'' \rightarrow h_{3A,1A}'', w_V'' \rightarrow w_{3A,1A}''), \\
c_+^{3A,1A}(q^2) &= a_+(q^2) \text{ with } (m_1'' \rightarrow -m_1'', h_V'' \rightarrow h_{3A,1A}'', w_V'' \rightarrow w_{3A,1A}''), \\
c_-^{3A,1A}(q^2) &= a_-(q^2) \text{ with } (m_1'' \rightarrow -m_1'', h_V'' \rightarrow h_{3A,1A}'', w_V'' \rightarrow w_{3A,1A}'').
\end{aligned} \tag{4.14}$$

It should be cautious that the replacement of $m_1'' \rightarrow -m_1''$ should not be applied to m_1'' in w'' and h'' . These form factors can be expressed in the $P_1^{3/2}$ and $P_1^{1/2}$ basis by using Eq. (3.4). For further details, the reader is referred to the earlier work [1].

C. Form-factor momentum dependence and numerical results

Because of the condition $q^+ = 0$ we have imposed during the course of calculation, form factors are known only for spacelike momentum transfer $q^2 = -q_\perp^2 \leq 0$, whereas only the timelike form factors are relevant for the physical decay processes. It has been proposed in [18] to recast the form factors as explicit functions of q^2 in the spacelike region and then analytically continue them to the timelike region. Another approach is to construct a double spectral representation for form factors at $q^2 < 0$ and then analytically continue it to $q^2 > 0$ region [58]. It has been shown recently that, within a specific model, form factors obtained directly from the timelike region (with $q^+ > 0$) are identical to the ones obtained by the analytic continuation from the spacelike region [59].

In principle, form factors at $q^2 > 0$ can be evaluated directly in the frame where the momentum transfer is purely longitudinal, i.e., $q_\perp = 0$, so that $q^2 = q^+q^-$ covers the entire range of momentum transfer [12]. The price one has to pay is that, besides the conventional valence-quark contribution, one must also consider the non-valence configuration (or the so-called Z -graph) arising from quark-pair creation from the vacuum. However, a reliable way of estimating the Z -graph contribution is still lacking unless one works in a specific model, for example, the one advocated in [59]. Fortunately, this additional non-valence contribution vanishes in the frame where the momentum transfer is purely transverse i.e., $q^+ = 0$.

To proceed we find that, except for the form factor V_2 to be discussed below, the momentum dependence of form factors in the spacelike region can be well parameterized and reproduced in the following three-parameter form:

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_{B(D)}^2) + b(q^2/m_{B(D)}^2)^2}, \quad (4.15)$$

for $P \rightarrow M$ transitions, where F stands for the relevant form factors appearing in these transitions.

The parameters a , b and $F(0)$ are first determined in the spacelike region. We then employ this parametrization to determine the physical form factors at $q^2 \geq 0$. In practice, the parameters a , b and $F(0)$ are obtained by performing a 5-parameter fit to the form factors in the range $-20 \text{ GeV}^2 \leq q^2 \leq 0$ for B decays and $-10 \text{ GeV}^2 \leq q^2 \leq 0$ for D decays. All $P \rightarrow M$ form factors are calculated at five q^2 values given below:

- a) for the charm sector: $q^2 = -0.01, -0.1, -1.0, -5.0, -10.0 \text{ GeV}^2$,
- b) for the bottom sector: $q^2 = -0.01, -0.1, -5.0, -10.0, -20.0 \text{ GeV}^2$.

These parameters are generally insensitive to the q^2 range to be fitted except for the form factor $V_2(q^2)$ in $B(D) \rightarrow {}^1P_1, P_1^{3/2}$ transitions. The obtained a and b coefficients are in most cases not far from unity as expected.

We have also analyzed the sensitivity of the form factors $F(0)$, and the slope parameters (a and b) to the uncertainties of β values. The form factors at $q^2 = 0$ are generally found to be less

sensitive to the variation in β values, whereas the corresponding parameters a and b are rather sensitive to the chosen range for β . Numerical results and discussion of these form factors and slope parameters (a and b) are presented in detail in the following section.

V. NUMERICAL RESULTS AND DISCUSSION

Equipped with the explicit expressions of the form factors $f_+(q^2), f_-(q^2)$ [Eqs. (4.5) and (4.6)] for $P \rightarrow P$ transitions, $g(q^2), f(q^2), a_+(q^2), a_-(q^2)$ [Eq. (4.7)] for $P \rightarrow V$ transitions, $u_+(q^2), u_-(q^2)$ [Eq. (4.13)] for $P \rightarrow S$ transitions, and $\ell(q^2), q(q^2), c_+(q^2), c_-(q^2)$ [Eq. (4.14)] for $P \rightarrow A$ transition, we now proceed to perform their numerical studies. In the earlier work, results for the form factors for $D(c\bar{u})$, $B(b\bar{u}) \rightarrow$ isovector (π like) and isospinor (K and D like) transition were calculated. In this work, we include isoscalar initial and final state mesons as well. Besides giving the updated results of these transitions due to the change in the strange quark mass, the variation of the β values and performing fit for five q^2 values, the $D_s, B_s \rightarrow P, V, S$, and A transition form factors are the main new results in this work.

In Tables IV–X, we present calculated form factors and their q^2 dependence, along with their allowed range due to uncertainties in β values of the initial and final mesons, for the $P(0^-) \rightarrow P(0^-), V(1^-), S(0^+), A(1^+ {}^3P_1)$, and $A(1^+ {}^1P_1)$ transitions of the charmed D, D_s and bottom B, B_s mesons. In calculations, we have taken the meson masses from the Particle Data Group [20]. Taking the natural flavor basis for isoscalar states of all the mesons (M), i.e., $M_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $M_s = (s\bar{s})$, we use the following masses (in GeV): $m_{\eta_q} = 0.741$ and $m_{\eta_s} = 0.802$ for pseudoscalar mesons taken from an analysis given in [56], $m_{f_{1q}} = 1.283$ and $m_{f_{1s}} = 1.425$ for the axial-vector (1^{++}) nonet, $m_{h_{1q}} = 1.242$ and $m_{h_{1s}} = 1.314$ for the other axial-vector (1^{+-}) case, and $m_{f_{0q}} = 1.474$ and $m_{f_{0s}} = 1.5$ for the scalar (0^{++}) mesons, based on the respective mixing schemes described in Sec. III. Form factors for transitions to the physical isosinglet diagonal states can be obtained from the Tables by including suitable Clebsch-Gordan coefficients. For instance,

$$\begin{aligned} A^{B_s f_1(1420)} &= -\sin \alpha_{f_1} A^{B_s f_{1s}}, & A^{B_s f_1(1285)} &= \cos \alpha_{f_1} A^{B_s f_{1s}}, \\ A^{B f_1(1420)} &= \frac{1}{\sqrt{2}} \cos \alpha_{f_1} A^{B f_{1q}}, & A^{B f_1(1285)} &= \frac{1}{\sqrt{2}} \sin \alpha_{f_1} A^{B f_{1q}}, \end{aligned} \quad (5.1)$$

where α_{f_1} has already been defined in Sec. III. The factor $\sqrt{2}$ appears for the $B \rightarrow f_1$ form factors, since either $u\bar{u}$ or $d\bar{d}$ component of f_{1q} can be transitioned from B meson via the appropriate weak current. Similarly, only the $s\bar{s}$ components of these mesons can be transitioned from the B_s meson. So the size of these corresponding form factors for physical isoscalar diagonal states gets reduced by the Clebsch-Gordan coefficients. Similar procedure can be adopted for transitions to the isosinglet diagonal states in other multiplets.

In these tables, two sets of uncertainties in the form factors, commonly denoted as $F(0)$, and their slope parameters (a and b) are given. The first and second sets of uncertainties shown in their values arise from the allowed uncertainties in the β parameter of the initial and final state meson, respectively. For the sake of clarity, it is mentioned here that the uncertainty shown as superscript (subscript) is due to the increase (decrease) in β of the corresponding meson. The obtained a and

b coefficients are in most cases not far from unity as expected. These parameters are generally insensitive to the q^2 range to be fitted, except for the form factor $V_2(q^2)$ in $B(D) \rightarrow {}^1P_1, P_1^{3/2}$ transitions. For these transitions, the corresponding parameters a and b are rather sensitive to the chosen range for q^2 , and quite larger than unity. This sensitivity is attributed to the fact that the form factor $V_2(q^2)$ approaches to zero at very large $-|q^2|$ where the three-parameter parametrization (4.15) becomes questionable. To overcome this difficulty, we follow [1] to fit this form factor to the following form:

$$F(q^2) = \frac{F(0)}{(1 - q^2/m_{B(D)}^2)[1 - a(q^2/m_{B(D)}^2) + b(q^2/m_{B(D)}^2)^2]}, \quad (5.2)$$

and achieve a substantial improvement. For example, we have $a = 2.18$ and $b = 6.08$ when $V_2^{BK_1P_1}$ is fitted to Eq. (4.15) and they become $a = 1.74$ and $b = 2.17$ (see Table IX) when the fit formula Eq. (5.2) is employed. It may be noted that we have considered parent meson constituted of heavy quark and light antiquark, since certain decay constants and form factors may change sign.

We make the following observations:

A. $P(0^-) \rightarrow P(0^-)$ Form Factors

- From Table IV, we notice that heavy-to-light form factors for the bottom mesons are smaller (around 0.3) than all the charmed meson form factors and the heavy-to-heavy bottom meson form factors, $F_{0,1}^{BD(B_s D_s)}$, which are around 0.7 or 0.8.
- We notice that the values of form factors at $q^2 = 0$ for B_s transitions are similar to the corresponding ones in B transitions. Therefore, flavor of the spectator quark does not seem to play a special role in affecting them. Particularly, we note the following for both $F_0(0)$ and $F_1(0)$: $F^{B_s D_s} = F^{BD}$, $F^{B_s K} \approx F^{B\pi}$, and $F^{B_s \eta_s} \approx F^{B\eta_q}$, where $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$, and η_s is pure $(s\bar{s})$ state. For the charm sector also, one may notice $F^{D_s K} = F^{D\pi} (\approx F^{BD})$ and $F^{D_s \eta_s} \approx F^{DK}$. However, the slope parameters, a and b , differ for these cases.
- Since the decay constants of pseudoscalar mesons are quite accurately determined, the errors on the β parameters are rather small. Correspondingly, the errors in the calculated form factors at $q^2 = 0$ are also very small. The same is true for the slope parameters except for a few cases, particularly for b , which may show large variation sometimes.
- Form factors ($F_1^{PP}(0)$ and $F_0^{PP}(0)$) usually tend to decrease (increase) with increasing (decreasing) β for initial meson, whereas they tend to increase (decrease) with increasing (decreasing) β for final meson. Only for B_s , the form factors show increasing (decreasing) trend for the initial as well as the final meson.
- Usually all the slope parameters are found to be positive. For the bottom sector, the slope parameters are larger than that for the charm sector. Particularly, the parameter b is much small (< 0.1 , if not zero) for $F_0^{PP}(0)$, except for $F_0^{B_s K}(0)$ and $F_0^{B_s \eta_s}(0)$ for which $b \approx 0.35$.

For $F_1^{B_s K}(0)$ and $F_1^{B_s \eta_s}(0)$, the parameter b is around 2 – 3 times larger than that for other cases.

- Slope parameters obtained using Eq. (4.15) generally tend to increase (decrease) with decrease (increase) in β for each of the initial and final mesons.
- According to the three-parameter parametrization Eq. (4.15), the dipole behavior corresponds to $b = (a/2)^2$, while $b = 0$ and $a \neq 0$ induces a monopole dependence. An inspection of Table IV indicates that form factors F_0^{PP} generally show a monopole behavior, and F_1^{PP} have a dipole behavior particularly for charmed meson transitions.

B. $P(0^-) \rightarrow V(1^-)$ Form Factors

- Like $P(0^-) \rightarrow P(0^-)$ form factors, we note the following behavior from Table V: $F^{B_s D_s^*} \approx F^{BD^*}$, $F^{B_s \phi} \approx F^{B\rho} \approx F^{B\omega}$, $F^{D_s \phi} \approx F^{DK^*}$ and $F^{D_s K^*} \approx F^{D\rho} \approx F^{D\omega}$, where F represents any of the form factors, V^{PV} , A_0^{PV} , A_1^{PV} or A_2^{PV} . However, the slope parameters show considerable differences for these cases.
- It is observed that heavy-to-light form factors for the bottom mesons are smaller (between 0.2 to 0.4) than all the charmed meson form factors and the heavy-to-heavy bottom meson form factors, $F^{BD^*(B_s D_s^*)}$, which lie between 0.6 to 1. We also notice the pattern $A_0^{PV} > A_1^{PV} > A_2^{PV}$ for all transitions, whereas $V^{PV} > A_0^{PV}$ for charmed meson and $B/B_s \rightarrow D/D_s$ transitions, but V^{PV} is slightly smaller than A_0^{PV} and remains greater than $A_{1,2}^{PV}$ for the heavy-to-light bottom transitions.

TABLE IV: Form factors of $P(0^-) \rightarrow P(0^-)$ transitions obtained in the covariant light-front model are fitted to the 3-parameter form Eq. (4.15). All the form factors are dimensionless.

F	$F(0)$	a	b	F	$F(0)$	a	b
$F_1^{D\pi}$	$0.66_{-0.01+0.00}^{+0.01-0.00}$	$1.19_{-0.01+0.00}^{+0.01+0.00}$	$0.35_{-0.03+0.00}^{+0.03+0.00}$	$F_0^{D\pi}$	$0.66_{-0.01+0.00}^{+0.01-0.00}$	$0.51_{-0.00+0.00}^{+0.00+0.00}$	$0.00_{-0.01+0.00}^{+0.01+0.00}$
$F_1^{D\eta_q}$	$0.71_{-0.00+0.01}^{+0.00-0.01}$	$1.13_{-0.01+0.02}^{+0.01+0.02}$	$0.27_{-0.02+0.02}^{+0.02+0.02}$	$F_0^{D\eta_q}$	$0.71_{-0.01+0.01}^{+0.00-0.01}$	$0.43_{-0.01+0.03}^{+0.01-0.03}$	$-0.01_{-0.01+0.00}^{+0.01-0.00}$
F_1^{DK}	$0.79_{-0.01+0.00}^{+0.01-0.00}$	$1.05_{-0.01+0.00}^{+0.01+0.00}$	$0.25_{-0.02+0.00}^{+0.02+0.00}$	F_0^{DK}	$0.79_{-0.01+0.00}^{+0.01-0.00}$	$0.47_{-0.00+0.01}^{+0.00-0.01}$	$-0.00_{-0.00+0.00}^{+0.00+0.00}$
$F_1^{D_s K}$	$0.66_{-0.00+0.00}^{+0.00-0.00}$	$1.11_{-0.00+0.00}^{+0.00+0.00}$	$0.48_{-0.03+0.01}^{+0.03+0.01}$	$F_0^{D_s K}$	$0.66_{-0.00+0.00}^{+0.00-0.00}$	$0.56_{-0.00+0.01}^{+0.00+0.01}$	$0.04_{-0.01+0.00}^{+0.01+0.00}$
$F_1^{D_s \eta_s}$	$0.76_{-0.00+0.02}^{+0.00-0.02}$	$1.02_{-0.01+0.01}^{+0.00+0.01}$	$0.40_{-0.02+0.05}^{+0.02+0.05}$	$F_0^{D_s \eta_s}$	$0.76_{-0.01+0.02}^{+0.00-0.03}$	$0.60_{-0.00+0.05}^{+0.00-0.06}$	$0.04_{-0.00+0.01}^{+0.00+0.01}$
$F_1^{B\pi}$	$0.25_{-0.00+0.00}^{+0.00-0.00}$	$1.70_{-0.03+0.00}^{+0.03+0.00}$	$0.90_{-0.05+0.00}^{+0.06+0.00}$	$F_0^{B\pi}$	$0.25_{-0.00+0.00}^{+0.00-0.00}$	$0.82_{-0.02+0.00}^{+0.02+0.00}$	$0.09_{-0.01+0.00}^{+0.02+0.00}$
$F_1^{B\eta_q}$	$0.29_{-0.00+0.01}^{+0.00-0.01}$	$1.63_{-0.02+0.02}^{+0.02+0.02}$	$0.74_{-0.04+0.04}^{+0.04+0.04}$	$F_0^{B\eta_q}$	$0.29_{-0.00+0.01}^{+0.00-0.01}$	$0.75_{-0.01+0.03}^{+0.01+0.03}$	$0.04_{-0.01+0.01}^{+0.01+0.01}$
F_1^{BK}	$0.34_{-0.00+0.00}^{+0.00-0.00}$	$1.60_{-0.02+0.00}^{+0.02+0.00}$	$0.73_{-0.04+0.01}^{+0.04+0.01}$	F_0^{BK}	$0.34_{-0.00+0.00}^{+0.00-0.00}$	$0.78_{-0.02+0.01}^{+0.02+0.01}$	$0.05_{-0.01+0.00}^{+0.01+0.00}$
F_1^{BD}	$0.67_{-0.00+0.01}^{+0.00-0.01}$	$1.22_{-0.01+0.01}^{+0.01+0.01}$	$0.36_{-0.01+0.01}^{+0.01+0.02}$	F_0^{BD}	$0.67_{-0.00+0.01}^{+0.00-0.01}$	$0.63_{-0.00+0.02}^{+0.00+0.02}$	$-0.01_{-0.01+0.00}^{+0.01+0.00}$
$F_1^{B_s K}$	$0.23_{-0.00+0.00}^{+0.00-0.00}$	$1.88_{-0.04+0.01}^{+0.04+0.01}$	$1.58_{-0.12+0.03}^{+0.14+0.03}$	$F_0^{B_s K}$	$0.23_{-0.00+0.00}^{+0.00-0.00}$	$1.05_{-0.03+0.01}^{+0.04+0.01}$	$0.35_{-0.05+0.02}^{+0.04+0.00}$
$F_1^{B_s \eta_s}$	$0.28_{-0.00+0.02}^{+0.00-0.02}$	$1.82_{-0.04+0.05}^{+0.04+0.05}$	$1.45_{-0.11+0.16}^{+0.13+0.18}$	$F_0^{B_s \eta_s}$	$0.28_{-0.00+0.02}^{+0.00-0.02}$	$1.07_{-0.03+0.06}^{+0.03+0.07}$	$0.32_{-0.04+0.06}^{+0.05+0.07}$
$F_1^{B_s D_s}$	$0.67_{-0.01+0.01}^{+0.00+0.01}$	$1.28_{-0.02+0.02}^{+0.02+0.02}$	$0.52_{-0.03+0.02}^{+0.03+0.02}$	$F_0^{B_s D_s}$	$0.67_{-0.01+0.01}^{+0.00+0.01}$	$0.69_{-0.01+0.02}^{+0.00+0.02}$	$0.07_{-0.01+0.01}^{+0.01+0.01}$

- Here also due to the reliability in fixing the β parameter for lighter vector mesons and the parent pseudoscalar mesons, the errors in the calculated form factors $F(0)$ are very small. In contrast, the slope parameters do show sensitivity to the variation in the β parameters specially in the bottom sector.
- Form factors, $V^{PV}(0)$, $A_0^{PV}(0)$, and $A_1^{PV}(0)$, for the charm sector usually tend to decrease (increase) with increasing (decreasing) β for initial meson, whereas they tend to increase (decrease) with increasing (decreasing) β for final meson. However, the form factor $A_2^{PV}(0)$ shows the opposite trend.
- For the bottom sector, form factors generally tend to increase (decrease) with increasing (decreasing) β for each of the initial and the final mesons.
- Slope parameters for all the cases are found to carry positive values. Both parameters (a and b) generally tend to increase (decrease) with decrease (increase) in β for each of the initial and final mesons.
- The parameters a is usually less sensitive to the β variation, whereas b is more sensitive to β values and may show large variation (10%) or even more sometimes for the bottom sector.
- Almost all the form factors for D as well as B are higher by (5 – 10)% than that obtained in the earlier work [1], whereas both slope parameter are reduced in magnitude. This could happen because now we perform 5-point fit for q^2 values.
- On comparison with $P \rightarrow P, V$ form factors obtained in the BSW model [57], the Melikhov-Stech (MS) model [60], QCD sum rule (QSR) [61], light-cone sum rules (LCSR) [62] and lattice calculations [63], it is pointed out that our predictions agree well with the available lattice results, and are most close to that of the MS model except for B_s transitions, which larger than our results. The LCSR and BSW model results are usually larger for $P \rightarrow V$ form factors for D and B transitions, however LCSR form factors for $B_s \rightarrow K^*$ transition match well with present work. The QSR calculations are generally lower than our results, except for $B \rightarrow K^*$ form factors which are higher than our predictions. Recently, $P \rightarrow V$ form factors for bottom mesons have also been calculated in the perturbative QCD approach [64], which are found to be lower than the values obtained in the present work.
- Experimentally, the form factors ratios $r_V \equiv V^{PV}(0)/A_1^{PV}(0)$ and $r_2 \equiv A_2^{PV}(0)/A_1^{PV}(0)$ are available for two semileptonic decays $D \rightarrow K^* \ell \nu$ and $D \rightarrow \phi \ell \nu$ [20]:

$$\begin{aligned}
r_V(D \rightarrow K^*) &= 1.62 \pm 0.08, & r_2(D \rightarrow K^*) &= 0.83 \pm 0.05, \\
r_V(D_s \rightarrow \phi) &= 1.82 \pm 0.08, & r_2(D_s \rightarrow \phi) &= 0.84 \pm 0.11.
\end{aligned} \tag{5.3}$$

Our predictions $r_2(D \rightarrow K^*) = 0.83$ and $r_2(D_s \rightarrow \phi) = 0.86$, agree well for both the decays, whereas $r_V(D \rightarrow K^*) = 1.36$ and $r_V(D_s \rightarrow \phi) = 1.42$ are lower than the corresponding experimental values.

C. $P(0^-) \rightarrow S(0^+)$ Form Factors

- It has been discussed in Sec. III that there are two sets of scalar mesons: the light scalar nonet formed by $\sigma(600)$, $\kappa(800)$, $f_0(980)$ and $a_0(980)$; and the heavy scalar nonet contains $a_0(1450)$, $K_0^*(1430)$, $f_0(1370)$ and $f_0(1500)/f_0(1710)$. Though their underlying quark structure is still controversial, the present experimental data seem to provide a consistent picture that light scalar mesons below or near 1 GeV can be described by the $q\bar{q}q\bar{q}$ states, while scalars above 1 GeV form a conventional $q\bar{q}$ with possible mixing with glueball states. In this work, we have calculated the form factors involving heavy scalar mesons, taking $f_0(1710)$ to be primarily a glueball, and $f_0(1500)$ and $f_0(1370)$ to be the SU(3) states as described in Sec. III.
- From Table VI, we notice that all the form factors for charmed mesons are around 0.5-0.6

TABLE V: Form factors of $P(0^-) \rightarrow V(1^-)$ transitions obtained in the covariant light-front model are fitted to the 3-parameter form Eq. (4.15). All the form factors are dimensionless.

F	$F(0)$	a	b	F	$F(0)$	a	b
$V^{D\rho}$	$0.88_{-0.02+0.01}^{+0.01-0.01}$	$1.23_{-0.01-0.00}^{+0.01+0.00}$	$0.40_{-0.03-0.01}^{+0.04+0.01}$	$A_0^{D\rho}$	$0.69_{-0.01+0.01}^{+0.01-0.01}$	$1.08_{-0.02-0.00}^{+0.01+0.00}$	$0.45_{-0.03-0.01}^{+0.03+0.01}$
$A_1^{D\rho}$	$0.60_{-0.00+0.00}^{+0.00-0.01}$	$0.46_{-0.02-0.01}^{+0.02+0.01}$	$0.01_{-0.00-0.00}^{+0.00+0.00}$	$A_2^{D\rho}$	$0.47_{-0.00+0.00}^{+0.00-0.00}$	$0.89_{-0.00+0.02}^{+0.00-0.02}$	$0.23_{-0.02-0.01}^{+0.02+0.01}$
$V^{D\omega}$	$0.85_{-0.02+0.01}^{+0.01-0.01}$	$1.24_{-0.01-0.00}^{+0.01+0.00}$	$0.45_{-0.04-0.01}^{+0.04+0.01}$	$A_0^{D\omega}$	$0.64_{-0.01+0.01}^{+0.01-0.01}$	$1.08_{-0.02+0.00}^{+0.01-0.00}$	$0.50_{-0.04-0.01}^{+0.04+0.01}$
$A_1^{D\omega}$	$0.58_{-0.01+0.00}^{+0.00-0.00}$	$0.49_{-0.02-0.01}^{+0.02+0.01}$	$0.02_{-0.00-0.00}^{+0.01+0.00}$	$A_2^{D\omega}$	$0.49_{-0.00+0.00}^{+0.00-0.00}$	$0.95_{-0.00+0.01}^{+0.00-0.01}$	$0.28_{-0.02-0.01}^{+0.02+0.01}$
V^{DK^*}	$0.98_{-0.02+0.01}^{+0.02-0.01}$	$1.10_{-0.02-0.00}^{+0.02+0.00}$	$0.32_{-0.03-0.01}^{+0.03+0.01}$	$A_0^{DK^*}$	$0.78_{-0.01+0.01}^{+0.01-0.01}$	$1.01_{-0.02-0.00}^{+0.02+0.00}$	$0.34_{-0.03-0.01}^{+0.03+0.01}$
$A_1^{DK^*}$	$0.72_{-0.01+0.01}^{+0.01-0.01}$	$0.45_{-0.02-0.01}^{+0.02+0.01}$	$0.01_{-0.00-0.00}^{+0.00+0.00}$	$A_2^{DK^*}$	$0.60_{-0.00+0.00}^{+0.00-0.00}$	$0.89_{-0.01-0.01}^{+0.00+0.01}$	$0.21_{-0.02-0.01}^{+0.02+0.01}$
$V^{D_s K^*}$	$0.87_{-0.01+0.01}^{+0.01-0.01}$	$1.13_{-0.01-0.00}^{+0.00+0.00}$	$0.69_{-0.04-0.02}^{+0.05+0.03}$	$A_0^{D_s K^*}$	$0.61_{-0.00+0.01}^{+0.00-0.01}$	$0.90_{-0.01+0.02}^{+0.01+0.01}$	$0.87_{-0.04-0.03}^{+0.05+0.01}$
$A_1^{D_s K^*}$	$0.56_{-0.00+0.01}^{+0.00-0.01}$	$0.59_{-0.01-0.01}^{+0.01+0.01}$	$0.08_{-0.01-0.00}^{+0.01+0.01}$	$A_2^{D_s K^*}$	$0.46_{-0.00+0.00}^{+0.00-0.00}$	$0.90_{-0.01+0.01}^{+0.01-0.01}$	$0.43_{-0.02-0.02}^{+0.02+0.02}$
$V^{D_s \phi}$	$0.98_{-0.01+0.00}^{+0.01-0.00}$	$1.04_{-0.00+0.00}^{+0.00-0.00}$	$0.54_{-0.03-0.00}^{+0.04+0.01}$	$A_0^{D_s \phi}$	$0.72_{-0.01+0.00}^{+0.01-0.00}$	$0.92_{-0.00+0.00}^{+0.00-0.00}$	$0.62_{-0.03-0.00}^{+0.04+0.01}$
$A_1^{D_s \phi}$	$0.69_{-0.00+0.00}^{+0.00-0.00}$	$0.56_{-0.02-0.00}^{+0.02+0.00}$	$0.07_{-0.01-0.00}^{+0.01+0.00}$	$A_2^{D_s \phi}$	$0.59_{-0.00+0.00}^{+0.00-0.00}$	$0.90_{-0.00+0.00}^{+0.00-0.00}$	$0.38_{-0.02-0.00}^{+0.02+0.00}$
$V^{B\rho}$	$0.29_{-0.00+0.01}^{+0.00-0.01}$	$1.77_{-0.03-0.01}^{+0.03+0.01}$	$1.06_{-0.06-0.03}^{+0.07+0.03}$	$A_0^{B\rho}$	$0.32_{-0.00+0.01}^{+0.00-0.01}$	$1.67_{-0.03-0.01}^{+0.03+0.03}$	$1.01_{-0.04-0.02}^{+0.05+0.02}$
$A_1^{B\rho}$	$0.24_{-0.00+0.00}^{+0.00-0.00}$	$0.86_{-0.03-0.01}^{+0.03+0.01}$	$0.15_{-0.02-0.01}^{+0.02+0.01}$	$A_2^{B\rho}$	$0.22_{-0.00+0.00}^{+0.00-0.00}$	$1.56_{-0.02-0.02}^{+0.02+0.02}$	$0.85_{-0.05-0.03}^{+0.05+0.03}$
$V^{B\omega}$	$0.27_{-0.00+0.00}^{+0.00-0.00}$	$1.81_{-0.03-0.01}^{+0.03+0.01}$	$1.18_{-0.07-0.02}^{+0.08+0.02}$	$A_0^{B\omega}$	$0.28_{-0.00+0.01}^{+0.00-0.01}$	$1.62_{-0.05+0.07}^{+0.11+0.09}$	$1.22_{-0.15-0.11}^{+0.03-0.08}$
$A_1^{B\omega}$	$0.23_{-0.00+0.00}^{+0.00-0.00}$	$0.91_{-0.03-0.01}^{+0.03+0.01}$	$0.18_{-0.02-0.01}^{+0.02+0.01}$	$A_2^{B\omega}$	$0.21_{-0.00+0.00}^{+0.00-0.00}$	$1.62_{-0.03-0.01}^{+0.03+0.01}$	$0.97_{-0.06-0.03}^{+0.06+0.02}$
V^{BK^*}	$0.36_{-0.00+0.01}^{+0.00-0.01}$	$1.69_{-0.03-0.01}^{+0.03+0.01}$	$0.95_{-0.06-0.02}^{+0.06+0.02}$	$A_0^{BK^*}$	$0.38_{-0.00+0.01}^{+0.00-0.01}$	$1.61_{-0.03-0.01}^{+0.03+0.01}$	$0.89_{-0.04-0.02}^{+0.05+0.02}$
$A_1^{BK^*}$	$0.31_{-0.00+0.00}^{+0.00-0.00}$	$0.84_{-0.03-0.01}^{+0.03+0.01}$	$0.12_{-0.02-0.01}^{+0.02+0.01}$	$A_2^{BK^*}$	$0.28_{-0.00+0.00}^{+0.00-0.00}$	$1.53_{-0.02-0.01}^{+0.02+0.01}$	$0.79_{-0.04-0.02}^{+0.05+0.02}$
V^{BD^*}	$0.77_{-0.01+0.02}^{+0.00-0.03}$	$1.25_{-0.02-0.03}^{+0.02+0.02}$	$0.38_{-0.02-0.03}^{+0.02+0.03}$	$A_0^{BD^*}$	$0.68_{-0.00+0.04}^{+0.00-0.04}$	$1.21_{-0.02-0.03}^{+0.02+0.02}$	$0.36_{-0.02-0.03}^{+0.02+0.03}$
$A_1^{BD^*}$	$0.65_{-0.00+0.02}^{+0.00-0.02}$	$0.60_{-0.01-0.03}^{+0.01+0.02}$	$0.00_{-0.00-0.01}^{+0.01+0.01}$	$A_2^{BD^*}$	$0.61_{-0.00+0.01}^{+0.00-0.01}$	$1.12_{-0.01-0.05}^{+0.01+0.04}$	$0.31_{-0.01-0.04}^{+0.01+0.04}$
$V^{B_s K^*}$	$0.23_{-0.00+0.01}^{+0.00-0.01}$	$2.03_{-0.04-0.01}^{+0.04+0.01}$	$2.27_{-0.20-0.07}^{+0.22+0.08}$	$A_0^{B_s K^*}$	$0.25_{-0.00+0.01}^{+0.00-0.01}$	$1.95_{-0.04-0.01}^{+0.04+0.01}$	$2.20_{-0.16-0.07}^{+0.18+0.08}$
$A_1^{B_s K^*}$	$0.19_{-0.00+0.00}^{+0.00-0.01}$	$1.24_{-0.05-0.02}^{+0.05+0.02}$	$0.62_{-0.07-0.03}^{+0.09+0.03}$	$A_2^{B_s K^*}$	$0.16_{-0.00+0.00}^{+0.00-0.00}$	$1.83_{-0.04-0.02}^{+0.04+0.02}$	$1.85_{-0.15-0.07}^{+0.17+0.08}$
$V^{B_s \phi}$	$0.29_{-0.00+0.00}^{+0.00-0.00}$	$1.95_{-0.04-0.00}^{+0.04+0.00}$	$1.98_{-0.17-0.02}^{+0.19+0.02}$	$A_0^{B_s \phi}$	$0.31_{-0.00+0.00}^{+0.00-0.00}$	$1.87_{-0.02-0.00}^{+0.04+0.00}$	$1.87_{-0.31-0.02}^{+0.16+0.02}$
$A_1^{B_s \phi}$	$0.25_{-0.01+0.00}^{+0.00-0.00}$	$1.20_{-0.05-0.01}^{+0.05+0.01}$	$0.54_{-0.06-0.02}^{+0.07+0.01}$	$A_2^{B_s \phi}$	$0.22_{-0.01+0.00}^{+0.00-0.00}$	$1.79_{-0.04-0.00}^{+0.04+0.00}$	$1.67_{-0.13-0.02}^{+0.15+0.02}$
$V^{B_s D_s^*}$	$0.75_{-0.00+0.03}^{+0.00-0.04}$	$1.37_{-0.03-0.05}^{+0.03+0.04}$	$0.67_{-0.05-0.08}^{+0.05+0.10}$	$A_0^{B_s D_s^*}$	$0.66_{-0.00+0.04}^{+0.00-0.05}$	$1.33_{-0.03-0.04}^{+0.03+0.04}$	$0.63_{-0.04-0.08}^{+0.05+0.10}$
$A_1^{B_s D_s^*}$	$0.62_{-0.00+0.02}^{+0.00-0.03}$	$0.76_{-0.03-0.05}^{+0.03+0.05}$	$0.13_{-0.02-0.03}^{+0.02+0.04}$	$A_2^{B_s D_s^*}$	$0.57_{-0.00+0.00}^{+0.00-0.01}$	$1.25_{-0.02-0.07}^{+0.02+0.07}$	$0.56_{-0.04-0.09}^{+0.04+0.11}$

where as all the bottom meson form factors lie 0.25 to 0.30, and thus are roughly half of the charmed meson form factors. Particularly, we note the following patterns: $F^{B_s K_0^*} = F^{Ba_0} = F^{B f_{0q}}$, $F^{B_s f_{0s}} \approx F^{BD_0^*} = F^{BK_0^*}$, and $F^{D_s f_{0s}} \approx F^{Da_0} = F^{D f_{0q}}$, where $f_{0q} = (u\bar{u} + d\bar{d})/\sqrt{2}$, and f_{0s} is a pure $(s\bar{s})$ state. Here too, the slope parameters show considerable differences among for the related form factors.

- Both of the form factors $F_{1,0}^{PS}(0)$ decrease (increase) with increasing (decreasing) β for initial meson, whereas they increase (decrease) with increasing (decreasing) β for final meson.
- The slope parameters b for both the form factors $F_{1,0}^{PS}$, and a for F_1^{PS} are found to be positive. For F_0^{PS} form factor, a turns out to be negative when charmed mesons appear in either initial or final state.
- For the bottom sector, the slope parameters are larger than that for the charm sector. The parameter b is generally much small (< 0.1) for $F_0^{PS}(0)$, except for $F_0^{B_s \rightarrow S(0^+)}$ and $F_0^{BD^*}$, for which b could be as big as 0.45.
- For transitions of the charmed mesons, the slope parameters for the form factor F_0^{PS} are more sensitive to change in β for each of the initial and final mesons. However, these are less sensitive for F_1^{PS} .
- Slope parameters (except for the case of negative a) show an increase (decrease) with decrease (increase) in β for each of the initial and final mesons.
- No significant change is found in the form factors, though a and b are slightly lowered than their values obtained in the earlier work [1]. Based on the light-cone sum rules, Chernyak [44] has estimated the $F_{1,0}^{Ba_0(1450)}(0) = 0.46$, while our result is 0.25 and is similar to the $B \rightarrow \pi$ form factor at $q^2 = 0$.
- On comparison of the $P \rightarrow S$ and $P \rightarrow P$ form factors, we notice $F^{D \rightarrow S} < F^{D \rightarrow P}$ for the same flavor content of the final state mesons. For the bottom sector, $F^{BD_0^*} < F^{BD}$ and $F^{B_s D_{s0}^*} < F^{B_s D_s}$, for heavy-to-heavy transitions, while $F^{B \rightarrow S} \approx F^{B \rightarrow P}$ for heavy-to-light transitions. It has been pointed out before [1] that the suppression of the $B \rightarrow D_0^*$ form factor relative to that of $B \rightarrow D$ is supported by experiment.
- An inspection of Table VI indicates that similar to the $P \rightarrow P$ transitions, the form factors F_0^{PS} generally show a monopole behavior, and F_1^{PS} have a dipole behavior particularly for charmed meson transitions. In general, form factors for $P \rightarrow S$ transitions increase slowly with q^2 compared to that for $P \rightarrow P$ ones.

D. $P(0^-) \rightarrow A(1^+ : {}^3P_1)$ Form Factors

- In Tables VII and IX, we have given heavy-to-light form factors involving axial vector nonet mesons, whereas heavy to heavy form factors are separately presented in Table X with the final state charmed mesons being taken as the heavy quark spin basis.

- From Table VII, all the form factors are found to be positive for the bottom as well as charm sectors. We also notice the following pattern: $V_1^{PA} > V_0^{PA} > A^{PA} > V_2^{PA}$ for the charmed mesons and $V_1^{PA} > A^{PA} > V_2^{PA} > V_0^{PA}$ for the bottom mesons.
- Numerically speaking, the form factor $A^{PA}(0)$ for the bottom transitions is generally around 0.25, and it is larger than that for charmed meson transition for which it lies close 0.16. Similar behavior is observed for $V_2^{PA}(0)$, which is < 0.1 for the charm sector, where as it lies around 0.2 for the bottom transitions. In contrast, the form factor $V_1^{PA}(0)$, lying around 0.4 for the bottom sector, is significantly smaller than that for the charm sector, where its value lies between 1.4 to 1.8. Also $V_0^{PA}(0)$ for the bottom transitions is roughly half of its value for charmed meson transitions.
- The form factors are not very sensitive to the variation chosen for the β parameters. However, they generally tend to decrease (increase) with increasing (decreasing) β for initial meson, whereas they increase (decrease) with increasing (decreasing) β for final meson.
- All the slope parameters, except for V_1^{PA} and V_2^{PA} for the charmed meson transitions, are found to be positive as per the definition given in Eq. (4.15). For the bottom sector, the slope parameters are significantly larger than that for the charm sector.
- Slope parameters a and b for A^{PA} , and V_1^{PA} are generally less sensitive to β variation, but for V_0^{PA} and V_2^{PA} they could show more sensitivity (even up to 20%) to change in β values.
- Generally no large change occurs in the form factors obtained in the earlier work [1], though slope parameters often show some changes.⁸ This could happen since we now perform 5-point

TABLE VI: Form factors of $P(0^-) \rightarrow S(0^+)$ transitions obtained in the covariant light-front model are fitted to the 3-parameter form Eq. (4.15). All the form factors are dimensionless.

F	$F(0)$	a	b	F	$F(0)$	a	b
$F_1^{Da_0}$	$0.51_{-0.01+0.01}^{+0.01-0.02}$	$1.06_{-0.02-0.02}^{+0.01+0.01}$	$0.24_{-0.02-0.02}^{+0.02+0.02}$	$F_0^{Da_0}$	$0.51_{-0.01+0.02}^{+0.01-0.02}$	$-0.04_{-0.06+0.07}^{+0.07-0.06}$	$0.02_{-0.02+0.01}^{+0.02-0.02}$
$F_1^{Df_{0q}}$	$0.51_{-0.01+0.03}^{+0.01-0.04}$	$1.06_{-0.02-0.04}^{+0.01+0.03}$	$0.24_{-0.02-0.04}^{+0.02+0.04}$	$F_0^{Df_{0q}}$	$0.51_{-0.01+0.04}^{+0.01-0.05}$	$-0.04_{-0.06+0.19}^{+0.07-0.13}$	$0.02_{-0.02+0.03}^{+0.02-0.04}$
$F_1^{DK_0^*}$	$0.47_{-0.01+0.02}^{+0.01-0.02}$	$0.94_{-0.02-0.01}^{+0.02+0.01}$	$0.19_{-0.02-0.01}^{+0.02+0.02}$	$F_0^{DK_0^*}$	$0.47_{-0.01+0.02}^{+0.01-0.03}$	$-0.31_{-0.04+0.05}^{+0.04-0.04}$	$0.08_{-0.01+0.01}^{+0.01-0.01}$
$F_1^{DsK_0^*}$	$0.55_{-0.01+0.03}^{+0.01-0.02}$	$1.02_{-0.01-0.01}^{+0.00+0.01}$	$0.38_{-0.02-0.04}^{+0.02+0.06}$	$F_0^{DsK_0^*}$	$0.55_{-0.01+0.03}^{+0.01-0.01}$	$-0.04_{-0.02+0.07}^{+0.02-0.05}$	$0.05_{-0.01+0.01}^{+0.01-0.01}$
$F_1^{Dsf_{0s}}$	$0.52_{-0.01+0.01}^{+0.01-0.00}$	$0.91_{-0.01-0.00}^{+0.01+0.00}$	$0.29_{-0.02-0.01}^{+0.02+0.01}$	$F_0^{Dsf_{0s}}$	$0.52_{-0.01+0.01}^{+0.01-0.01}$	$-0.34_{-0.01+0.01}^{+0.01-0.01}$	$0.10_{-0.01+0.00}^{+0.01-0.00}$
$F_1^{Ba_0}$	$0.25_{-0.00+0.01}^{+0.00-0.01}$	$1.53_{-0.03-0.01}^{+0.03+0.01}$	$0.64_{-0.04-0.04}^{+0.05+0.04}$	$F_0^{Ba_0}$	$0.25_{-0.00+0.01}^{+0.00-0.01}$	$0.54_{-0.01-0.03}^{+0.01+0.03}$	$0.01_{-0.01-0.00}^{+0.02+0.00}$
$F_1^{Bf_{0q}}$	$0.25_{-0.00+0.03}^{+0.00-0.03}$	$1.53_{-0.03-0.03}^{+0.03+0.03}$	$0.64_{-0.04-0.08}^{+0.05+0.11}$	$F_0^{Bf_{0q}}$	$0.25_{-0.00+0.03}^{+0.00-0.03}$	$0.54_{-0.01+0.07}^{+0.01+0.06}$	$0.01_{-0.01-0.00}^{+0.02+0.01}$
$F_1^{BK_0^*}$	$0.27_{-0.01+0.01}^{+0.01-0.02}$	$1.43_{-0.03-0.01}^{+0.03+0.01}$	$0.52_{-0.04-0.03}^{+0.04+0.04}$	$F_0^{BK_0^*}$	$0.27_{-0.01+0.01}^{+0.01-0.02}$	$0.32_{-0.01-0.02}^{+0.01+0.03}$	$0.05_{-0.01+0.00}^{+0.01-0.00}$
$F_1^{BD_0^*}$	$0.27_{-0.01+0.03}^{+0.01-0.03}$	$1.08_{-0.04+0.03}^{+0.04-0.07}$	$0.23_{-0.02-0.00}^{+0.02+0.00}$	$F_0^{BD_0^*}$	$0.27_{-0.01+0.03}^{+0.01-0.03}$	$-0.48_{-0.02+0.01}^{+0.02-0.03}$	$0.36_{-0.03+0.03}^{+0.03-0.03}$
$F_1^{BsK_0^*}$	$0.25_{-0.00+0.02}^{+0.00-0.02}$	$1.75_{-0.04-0.04}^{+0.04+0.05}$	$1.33_{-0.12-0.12}^{+0.14+0.18}$	$F_0^{BsK_0^*}$	$0.25_{-0.00+0.02}^{+0.00-0.02}$	$0.74_{-0.03-0.06}^{+0.03+0.07}$	$0.24_{-0.04-0.03}^{+0.04+0.05}$
$F_1^{Bsf_{0s}}$	$0.28_{-0.00+0.01}^{+0.00-0.01}$	$1.64_{-0.04-0.01}^{+0.04+0.01}$	$1.07_{-0.10-0.04}^{+0.11+0.04}$	$F_0^{Bsf_{0s}}$	$0.28_{-0.00+0.01}^{+0.00-0.01}$	$0.52_{-0.03-0.02}^{+0.03+0.02}$	$0.20_{-0.03-0.01}^{+0.03+0.01}$
$F_1^{BsD_0^*}$	$0.30_{-0.02+0.03}^{+0.02-0.03}$	$1.18_{-0.06+0.01}^{+0.06-0.04}$	$0.51_{-0.05-0.05}^{+0.06+0.05}$	$F_0^{BsD_0^*}$	$0.30_{-0.02+0.03}^{+0.02-0.04}$	$-0.47_{-0.01-0.02}^{+0.01+0.01}$	$0.45_{-0.02+0.01}^{+0.02+0.00}$

⁸ Form factor $A^{DK_1(^3P_1)}(0) = 0.98$ given in the earlier [1] is erroneous, and should be replaced with 0.15.

fit for q^2 values. We find that the form factors V_0^{PA} for $B \rightarrow a_1$ transition has marginally increased to 0.14.

- There are several existing model calculations for $B \rightarrow A$ form factors: the ISGW2 model [11], the constituent quark-meson model (CQM) [65], the QCD sum rules (QSR) [66], light cone sum rules (LCSR) [67], and more recently the perturbative QCD (pQCD) approach [64]. For the sake of comparison, results for $B \rightarrow a_1$ transition form factors are given in Table VIII for these approaches, which show quite significant differences since these approaches differ in their treatment of dynamics of form factors. For example, $V_0^{Ba_1} = 1.20$, obtained in the quark-meson model and 1.01 in the ISGW2 model, are larger than the values obtained in other approaches. If $a_1(1260)$ behaves as the scalar partner of the ρ meson, $V_0^{Ba_1}$ is expected to be similar to $A_0^{B\rho}$, which is of order 0.3 at $q^2 = 0$. Therefore, it appears to us that a magnitude of order unity for $V_0^{Ba_1}$ as predicted by the ISGW2 model and CQM is very unlikely. In principle, the experimental measurements of $\bar{B}^0 \rightarrow a_1^\pm \pi^\mp$ will enable us to test the form factors $V_0^{Ba_1}$. The BaBar and Belle measurements [68, 69] of $\bar{B}^0 \rightarrow a_1^\pm \pi^\mp$ favors a value of $V_0^{Ba_1}(0) \approx 0.30$ [70], which is very close the LCSR result shown in Table VIII.

TABLE VII: Form factors of $P(0^-) \rightarrow A(1^{++})$ transitions obtained in the covariant light-front model are fitted to the 3-parameter form Eq. (4.15). All the form factors are dimensionless.

F	$F(0)$	a	b	F	$F(0)$	a	b
A^{Da_1}	$0.19^{+0.01+0.00}_{-0.01-0.01}$	$1.03^{+0.02+0.00}_{-0.03-0.01}$	$0.16^{+0.02+0.01}_{-0.02-0.01}$	$V_0^{Da_1}$	$0.32^{+0.00-0.00}_{-0.00-0.00}$	$0.96^{+0.00+0.00}_{-0.01-0.01}$	$0.43^{+0.07-0.01}_{-0.06+0.01}$
$V_1^{Da_1}$	$1.51^{+0.04+0.00}_{-0.04-0.01}$	$-0.06^{+0.01+0.02}_{-0.01-0.02}$	$0.04^{+0.00+0.00}_{-0.00-0.00}$	$V_2^{Da_1}$	$0.05^{+0.01-0.00}_{-0.01-0.00}$	$-0.02^{+0.07+0.00}_{-0.08-0.00}$	$0.12^{+0.00-0.01}_{-0.00+0.01}$
$A^{Df_{1q}}$	$0.18^{+0.01+0.01}_{-0.01-0.01}$	$1.03^{+0.02+0.00}_{-0.03-0.02}$	$0.16^{+0.02+0.03}_{-0.02-0.02}$	$V_0^{Df_{1q}}$	$0.34^{+0.00-0.00}_{-0.00-0.00}$	$0.97^{+0.01-0.00}_{-0.02-0.01}$	$0.39^{+0.06+0.00}_{-0.05+0.03}$
$V_1^{Df_{1q}}$	$1.75^{+0.04+0.02}_{-0.05-0.00}$	$-0.02^{+0.01+0.06}_{-0.01-0.05}$	$0.04^{+0.00-0.00}_{-0.00+0.01}$	$V_2^{Df_{1q}}$	$0.05^{+0.01-0.00}_{-0.01-0.00}$	$-0.02^{+0.07+0.01}_{-0.08-0.01}$	$0.12^{+0.00-0.01}_{-0.00+0.02}$
$A^{DK_{1A}}$	$0.15^{+0.01+0.01}_{-0.01-0.01}$	$0.89^{+0.03+0.00}_{-0.03-0.01}$	$0.12^{+0.02+0.01}_{-0.02-0.01}$	$V_0^{DK_{1A}}$	$0.28^{+0.00+0.00}_{-0.00+0.00}$	$0.84^{+0.01-0.01}_{-0.02-0.01}$	$0.39^{+0.06+0.00}_{-0.05+0.04}$
$V_1^{DK_{1A}}$	$1.60^{+0.05+0.01}_{-0.05-0.02}$	$-0.22^{+0.00+0.03}_{-0.00-0.03}$	$0.07^{+0.00-0.00}_{-0.00+0.00}$	$V_2^{DK_{1A}}$	$0.01^{+0.00-0.00}_{-0.00-0.00}$	$-0.83^{+0.15-0.03}_{-0.17+0.02}$	$0.24^{+0.04-0.01}_{-0.03+0.01}$
$A^{D_s K_{1A}}$	$0.19^{+0.01+0.01}_{-0.01-0.01}$	$0.99^{+0.01+0.01}_{-0.01-0.01}$	$0.28^{+0.02+0.04}_{-0.02-0.03}$	$V_0^{D_s K_{1A}}$	$0.29^{+0.00+0.00}_{-0.00+0.00}$	$0.72^{+0.05-0.09}_{-0.06+0.07}$	$0.87^{+0.09+0.08}_{-0.10-0.05}$
$V_1^{D_s K_{1A}}$	$1.68^{+0.03+0.00}_{-0.03-0.01}$	$-0.04^{+0.01+0.05}_{-0.01-0.04}$	$0.06^{+0.00-0.00}_{-0.00-0.00}$	$V_2^{D_s K_{1A}}$	$0.07^{+0.00-0.00}_{-0.00-0.00}$	$0.22^{+0.03+0.02}_{-0.03-0.01}$	$0.15^{+0.01+0.02}_{-0.01-0.01}$
$A^{D_s f_{1s}}$	$0.17^{+0.01+0.00}_{-0.01-0.00}$	$0.86^{+0.02+0.00}_{-0.02-0.00}$	$0.20^{+0.02+0.01}_{-0.02-0.01}$	$V_0^{D_s f_{1s}}$	$0.22^{+0.00+0.00}_{-0.00+0.00}$	$0.19^{+0.13-0.13}_{-0.17+0.10}$	$1.20^{+0.18-0.09}_{-0.15+0.11}$
$V_1^{D_s f_{1s}}$	$1.47^{+0.03+0.01}_{-0.03-0.01}$	$-0.29^{+0.01+0.01}_{-0.01-0.01}$	$0.09^{+0.00+0.00}_{-0.00-0.00}$	$V_2^{D_s f_{1s}}$	$0.03^{+0.00-0.00}_{-0.00-0.00}$	$-0.34^{+0.05+0.00}_{-0.05-0.00}$	$0.17^{+0.00-0.00}_{-0.00+0.00}$
A^{Ba_1}	$0.24^{+0.01+0.01}_{-0.01-0.01}$	$1.48^{+0.03+0.01}_{-0.03-0.01}$	$0.57^{+0.05+0.04}_{-0.04-0.03}$	$V_0^{Ba_1}$	$0.14^{+0.01+0.01}_{-0.01-0.01}$	$1.66^{+0.04+0.01}_{-0.04-0.01}$	$1.11^{+0.09+0.02}_{-0.08-0.03}$
$V_1^{Ba_1}$	$0.36^{+0.01+0.01}_{-0.01-0.01}$	$0.26^{+0.02+0.03}_{-0.02-0.02}$	$0.14^{+0.01+0.01}_{-0.01-0.01}$	$V_2^{Ba_1}$	$0.17^{+0.01+0.01}_{-0.01-0.01}$	$1.08^{+0.05+0.02}_{-0.05-0.02}$	$0.44^{+0.09+0.04}_{-0.03-0.03}$
$A^{Bf_{1q}}$	$0.24^{+0.01+0.02}_{-0.01-0.02}$	$1.48^{+0.03+0.03}_{-0.03-0.03}$	$0.57^{+0.05+0.10}_{-0.04-0.08}$	$V_0^{Bf_{1q}}$	$0.14^{+0.01+0.01}_{-0.01-0.01}$	$1.65^{+0.04+0.02}_{-0.04-0.02}$	$1.07^{+0.09+0.09}_{-0.08-0.06}$
$V_1^{Bf_{1q}}$	$0.37^{+0.01+0.03}_{-0.01-0.03}$	$0.27^{+0.02+0.06}_{-0.02-0.05}$	$0.13^{+0.01+0.02}_{-0.01-0.01}$	$V_2^{Bf_{1q}}$	$0.17^{+0.01+0.01}_{-0.01-0.01}$	$1.08^{+0.05+0.05}_{-0.05-0.04}$	$0.44^{+0.03+0.09}_{-0.03-0.07}$
$A^{BK_{1A}}$	$0.27^{+0.01+0.01}_{-0.01-0.01}$	$1.39^{+0.04+0.00}_{-0.04-0.01}$	$0.47^{+0.04+0.04}_{-0.04-0.03}$	$V_0^{BK_{1A}}$	$0.16^{+0.01+0.01}_{-0.01-0.01}$	$1.55^{+0.06-0.01}_{-0.03+0.01}$	$1.00^{+0.09+0.03}_{-0.10-0.02}$
$V_1^{BK_{1A}}$	$0.39^{+0.01+0.01}_{-0.01-0.01}$	$0.07^{+0.02+0.02}_{-0.02-0.02}$	$0.19^{+0.00+0.00}_{-0.00-0.00}$	$V_2^{BK_{1A}}$	$0.17^{+0.01+0.01}_{-0.01-0.01}$	$0.84^{+0.06-0.01}_{-0.06+0.01}$	$0.36^{+0.02+0.04}_{-0.02-0.03}$
$A^{B_s K_{1A}}$	$0.24^{+0.00+0.02}_{-0.00-0.02}$	$1.70^{+0.05+0.05}_{-0.05-0.04}$	$1.22^{+0.13+0.17}_{-0.12-0.12}$	$V_0^{B_s K_{1A}}$	$0.12^{+0.01+0.01}_{-0.01-0.01}$	$1.88^{+0.06+0.03}_{-0.05-0.02}$	$2.06^{+0.30+0.13}_{-0.25-0.08}$
$V_1^{B_s K_{1A}}$	$0.37^{+0.00+0.02}_{-0.00-0.02}$	$0.53^{+0.04+0.07}_{-0.04-0.06}$	$0.29^{+0.03+0.04}_{-0.03-0.03}$	$V_2^{B_s K_{1A}}$	$0.17^{+0.00+0.01}_{-0.00-0.01}$	$1.40^{+0.07+0.05}_{-0.07-0.04}$	$0.97^{+0.12+0.15}_{-0.10-0.10}$
$A^{B_s f_{1s}}$	$0.28^{+0.01+0.01}_{-0.01-0.01}$	$1.59^{+0.05+0.01}_{-0.05-0.01}$	$0.99^{+0.11+0.04}_{-0.10-0.03}$	$V_0^{B_s f_{1s}}$	$0.13^{+0.01+0.00}_{-0.01-0.00}$	$1.79^{+0.06-0.00}_{-0.05+0.00}$	$2.00^{+0.33+0.02}_{-0.25-0.01}$
$V_1^{B_s f_{1s}}$	$0.41^{+0.01+0.01}_{-0.01-0.01}$	$0.29^{+0.04+0.02}_{-0.04-0.02}$	$0.29^{+0.02+0.01}_{-0.02-0.01}$	$V_2^{B_s f_{1s}}$	$0.18^{+0.01+0.00}_{-0.01-0.00}$	$1.18^{+0.07+0.01}_{-0.07-0.01}$	$0.74^{+0.03+0.03}_{-0.08+0.03}$

E. $P(0^-) \rightarrow A(1^+ : {}^1P_1)$ Form Factors

- From Table IX, we find that the form factors, $A^{PA}(0)$, $V_0^{PA}(0)$ and $V_1^{PA}(0)$, are positive, where as $V_2^{PA}(0)$ is negative and small (around -0.1) for the bottom as well as charm sector, and follow the pattern: $V_1^{PA} > V_0^{PA} > A^{PA} > |V_2^{PA}|$ for the charmed mesons and $V_0^{PA} > V_1^{PA} > A^{PA} \geq |V_2^{PA}|$ for the bottom mesons. Numerically, form factors $A^{PA}(0)$ is generally around 0.1, where as $V_0^{PA}(0)$ lies close to 0.5 for all the cases. The form factor $V_1^{PA}(0)$, lying between 0.15 to 0.20, for the bottom sector is significantly smaller than that for the charm sector, where its value lies between 1.3 to 1.6.
- The form factors $A_0^{PA}(0)$ and $V_1^{PA}(0)$ usually increase (decrease) with increasing (decreasing) beta for initial meson as well as for final meson.
- The form factors $V_0^{PA}(0)$ and $V_3^{PA}(0)$ decrease (increase) in magnitude with increasing (decreasing) β for initial meson, and show the opposite trend for final mesons, i.e., these increase (decrease) with increasing (decreasing) β for the final state.
- All the slope parameters are found to be positive. For the bottom sector, the slope parameters are larger than that for the charm sector.
- Slope parameters a and b for A^{PA} , V_0^{PA} , and V_1^{PA} are less sensitive (a few %) to the variation in the β values. For V_2^{PA} form factor, the slope parameters show huge sensitivity to the change in beta values, even with assuming q^2 behavior given by Eq. (4.15). However, these are less sensitive for $B \rightarrow b_1/h_{1q}$ cases.
- No significant change is found in the form factors obtained in the earlier work [1], however, the slope parameters show difference.
- While comparing the form factors of heavy-to-light spin 1 meson transitions, we notice the following relations for the same flavor content of the mesons: $A^{PA(1^{++})}(0) > A^{PA(1^{+-})}(0)$, $V_1^{PA(1^{++})}(0) > V_1^{PA(1^{+-})}(0)$. But for $V_0^{PA}(0)$ form factors, we find $V_0^{PA(1^{++})}(0) < V_0^{PA(1^{+-})}(0)$. For $V_2^{PA}(0)$ form factors, we observe opposite behavior for the charmed and bottom mesons, i.e., $|V_2^{D,Ds \rightarrow A(1^{++})}(0)| < |V_2^{D,Ds \rightarrow A(1^{+-})}(0)|$, whereas $|V_2^{B,Bs \rightarrow A(1^{++})}(0)| > |V_2^{B,Bs \rightarrow A(1^{+-})}(0)|$.

TABLE VIII: Form factors of $B \rightarrow a_1$ transitions at maximum recoil ($q^2 = 0$). The results of CQM and QSR have been rescaled according to the form factor definition in Eq. (4.9)

$B \rightarrow a_1$	This work	ISGW2 [11]	CQM [65]	QSR [66]	LCSR [67]	pQCD [64]
A	$0.24_{-0.01}^{+0.01}$	0.21	0.09	0.41 ± 0.06	0.48 ± 0.09	$0.26_{-0.05}^{+0.06+0.00+0.03}$
V_0	$0.14_{-0.01}^{+0.01+0.01}$	1.01	1.20	0.23 ± 0.05	0.30 ± 0.05	$0.34_{-0.07}^{+0.07+0.01+0.08}$
V_1	$0.36_{-0.01}^{+0.01+0.01}$	0.54	1.32	0.68 ± 0.08	0.37 ± 0.07	$0.43_{-0.09}^{+0.10+0.01+0.05}$
V_2	$0.17_{-0.01}^{+0.01+0.01}$	-0.05	0.34	0.33 ± 0.03	0.42 ± 0.08	$0.13_{-0.03}^{+0.03+0.00+0.00}$

- While comparing the form factors of heavy-to-light vector and axial-vector mesons, we notice the following patterns: $V^{PV}(0) > A^{PA(1^{++})}(0)$ for the same flavor content of the mesons; $V_1^{PA(1^{+-})}(0) > A_1^{PV}(0)$ for charmed mesons and $A_1^{PV}(0) > V_1^{PA(1^{+-})}(0)$ for the bottom mesons. But for $V_0^{PA}(0)$ form factors, we find $V_0^{PA(1^{+-})}(0) < A_0^{PV}(0)$ for charmed mesons and $V_0^{PA(1^{++})}(0) < A_0^{PV}(0) < V_0^{PA(1^{+-})}(0)$ for the bottom mesons. We also observe that $A_2^{PV}(0)$ is higher than both $V_2^{D,Ds \rightarrow A(1^{++})}(0)$ as well as $|V_2^{D,Ds \rightarrow A(1^{+-})}(0)|$.

F. $B(0^-) \rightarrow D^{1/2}, D^{3/2}$ Form Factors

- From Tables X, we notice that most of the form factors are small and lie between 0.1 to 0.25, except $V_0(0)$ and $V_1(0)$ for the transitions emitting $P_1^{3/2}$ states, for which these lie between 0.5 to 0.6. In contrast with these, $B, B_s \rightarrow D, D_s$ form factors carry the highest values between 0.6 to 0.8.
- Slope parameters carry positive values except a for V_1 form factor. However, these parameters controlling q^2 behavior for V_2 form factor for transitions emitting $P_1^{3/2}$ states remains difficult to control in spite of choosing the q^2 dependence given in Eq. (4.15).
- Reverse changes occur in the form factors due to the variation in β values for initial and final mesons. Increase in β for initial (final) state meson tend to decrease (increase) the magnitude

TABLE IX: Form factors of $P(0^-) \rightarrow A(1^{+-})$ transitions obtained in the covariant light-front model are fitted to the 3-parameter form Eq. (4.15). All the form factors are dimensionless.

F	$F(0)$	a	b	F	$F(0)$	a	b
A^{Db_1}	$0.12^{+0.00+0.00}_{-0.00-0.02}$	$1.09^{+0.01+0.01}_{-0.03-0.02}$	$0.50^{+0.04+0.04}_{-0.01-0.00}$	$V_0^{Db_1}$	$0.50^{+0.01+0.02}_{-0.01-0.02}$	$0.98^{+0.02-0.01}_{-0.02-0.00}$	$0.26^{+0.00+0.03}_{-0.00-0.16}$
$V_1^{Db_1}$	$1.39^{+0.02+0.03}_{-0.02-0.04}$	$0.44^{+0.03+0.02}_{-0.01-0.01}$	$0.05^{+0.01+0.01}_{-0.02+0.01}$	$V_2^{Db_1}$	$-0.10^{+0.02-0.01}_{-0.02+0.01}$	$0.26^{+0.28-0.23}_{-0.02-0.03}$	$0.90^{+0.23+0.23}_{-0.23+0.23}$
$A^{Dh_{1q}}$	$0.11^{+0.00+0.00}_{-0.00-0.01}$	$1.09^{+0.01+0.01}_{-0.03-0.04}$	$0.50^{+0.04+0.10}_{-0.01-0.01}$	$V_0^{Dh_{1q}}$	$0.49^{+0.01+0.04}_{-0.01-0.05}$	$0.98^{+0.02-0.03}_{-0.02-0.02}$	$0.26^{+0.00+0.06}_{-0.00-0.06}$
$V_1^{Dh_{1q}}$	$1.42^{+0.02+0.06}_{-0.02-0.09}$	$0.44^{+0.03+0.04}_{-0.01-0.01}$	$0.05^{+0.01+0.02}_{-0.03-0.03}$	$V_2^{Dh_{1q}}$	$-0.10^{+0.02-0.02}_{-0.01+0.03}$	$0.26^{+0.28-0.24}_{-0.02-0.02}$	$0.90^{+0.46-0.33}_{-0.23+0.69}$
$A^{DK_{1B}}$	$0.10^{+0.00+0.00}_{-0.00-0.00}$	$0.98^{+0.01+0.01}_{-0.02-0.01}$	$0.37^{+0.03+0.04}_{-0.00-0.00}$	$V_0^{DK_{1B}}$	$0.48^{+0.01+0.02}_{-0.01-0.03}$	$0.94^{+0.02-0.02}_{-0.01+0.01}$	$0.22^{+0.00+0.03}_{-0.00-0.03}$
$V_1^{DK_{1B}}$	$1.58^{+0.02+0.03}_{-0.03-0.05}$	$0.31^{+0.02+0.02}_{-0.02-0.01}$	$0.04^{+0.00+0.01}_{-0.00-0.00}$	$V_2^{DK_{1B}}$	$-0.13^{+0.01-0.01}_{-0.01+0.01}$	$0.57^{+0.06-0.01}_{-0.04-0.01}$	$0.32^{+0.05-0.04}_{-0.03+0.06}$
$A^{D_s K_{1B}}$	$0.10^{+0.00+0.00}_{-0.00-0.00}$	$0.97^{+0.01+0.02}_{-0.01-0.16}$	$0.71^{+0.03+0.19}_{-0.01-0.01}$	$V_0^{D_s K_{1B}}$	$0.51^{+0.01+0.03}_{-0.01-0.04}$	$0.91^{+0.01+0.00}_{-0.01-0.03}$	$0.45^{+0.00-0.07}_{-0.00+0.10}$
$V_1^{D_s K_{1B}}$	$1.50^{+0.01+0.05}_{-0.01-0.07}$	$0.59^{+0.02+0.03}_{-0.02-0.02}$	$0.10^{+0.01+0.02}_{-0.01-0.01}$	$V_2^{D_s K_{1B}}$	$-0.12^{+0.01-0.01}_{-0.01+0.01}$	$0.68^{+0.02-0.02}_{-0.01+0.01}$	$0.36^{+0.01-0.05}_{-0.01+0.08}$
$A^{D_s h_{1s}}$	$0.10^{+0.00+0.00}_{-0.00-0.00}$	$0.93^{+0.00-0.00}_{-0.00-0.00}$	$0.51^{+0.03+0.02}_{-0.01-0.00}$	$V_0^{D_s h_{1s}}$	$0.57^{+0.01+0.01}_{-0.01-0.01}$	$0.89^{+0.01+0.00}_{-0.01-0.00}$	$0.37^{+0.00-0.02}_{-0.00+0.03}$
$V_1^{D_s h_{1s}}$	$1.43^{+0.01+0.02}_{-0.01-0.02}$	$0.46^{+0.02+0.01}_{-0.02-0.01}$	$0.07^{+0.01+0.00}_{-0.01+0.00}$	$V_2^{D_s h_{1s}}$	$-0.17^{+0.01-0.00}_{-0.01+0.00}$	$0.55^{+0.02-0.01}_{-0.01+0.01}$	$0.20^{+0.01-0.01}_{-0.00+0.01}$
A^{Bb_1}	$0.11^{+0.00+0.00}_{-0.00-0.01}$	$1.89^{+0.03-0.03}_{-0.03+0.03}$	$1.51^{+0.08-0.09}_{-0.09+0.10}$	$V_0^{Bb_1}$	$0.38^{+0.01+0.03}_{-0.01-0.03}$	$1.38^{+0.03+0.00}_{-0.03-0.01}$	$0.63^{+0.02-0.04}_{-0.03+0.04}$
$V_1^{Bb_1}$	$0.19^{+0.01+0.01}_{-0.01-0.01}$	$0.99^{+0.03-0.03}_{-0.03+0.04}$	$0.29^{+0.03-0.03}_{-0.03+0.03}$	$V_2^{Bb_1}$	$-0.02^{+0.01-0.01}_{-0.01+0.00}$	$1.11^{+20.4+0.33}_{-0.75-0.75}$	$7.76^{+59.7-1.82}_{-4.34+3.28}$
$A^{Bh_{1q}}$	$0.10^{+0.00+0.01}_{-0.00-0.01}$	$1.89^{+0.03-0.07}_{-0.03+0.07}$	$1.51^{+0.08-0.20}_{-0.09+0.25}$	$V_0^{Bh_{1q}}$	$0.37^{+0.01+0.07}_{-0.01-0.06}$	$1.37^{+0.03-0.00}_{-0.03-0.03}$	$0.62^{+0.02-0.08}_{-0.03+0.11}$
$V_1^{Bh_{1q}}$	$0.19^{+0.01+0.02}_{-0.01-0.02}$	$0.99^{+0.03-0.08}_{-0.03+0.08}$	$0.29^{+0.03-0.06}_{-0.03+0.09}$	$V_2^{Bh_{1q}}$	$-0.02^{+0.01-0.01}_{-0.01+0.01}$	$1.11^{+20.4+0.53}_{-0.75-3.74}$	$7.76^{+59.7-3.38}_{-4.35+13.7}$
$A^{BK_{1B}}$	$0.12^{+0.00+0.01}_{-0.00-0.01}$	$1.78^{+0.03-0.03}_{-0.03+0.04}$	$1.27^{+0.07-0.08}_{-0.07+0.11}$	$V_0^{BK_{1B}}$	$0.45^{+0.01+0.04}_{-0.01-0.04}$	$1.37^{+0.03-0.00}_{-0.03-0.01}$	$0.54^{+0.02-0.04}_{-0.02+0.05}$
$V_1^{BK_{1B}}$	$0.21^{+0.01+0.01}_{-0.01-0.01}$	$0.83^{+0.03-0.03}_{-0.03+0.04}$	$0.22^{+0.02-0.02}_{-0.02+0.03}$	$V_2^{BK_{1B}}$	$-0.05^{+0.01-0.01}_{-0.01+0.01}$	$1.74^{+0.08-0.03}_{-0.00+0.02}$	$2.17^{+1.09-0.23}_{-0.53+0.31}$
$A^{B_s K_{1B}}$	$0.08^{+0.00+0.01}_{-0.00-0.01}$	$2.06^{+0.04-0.04}_{-0.03+0.05}$	$2.57^{+0.20-0.23}_{-0.23+0.33}$	$V_0^{B_s K_{1B}}$	$0.38^{+0.01+0.04}_{-0.01-0.05}$	$1.64^{+0.04-0.03}_{-0.01+0.04}$	$1.25^{+0.07-0.13}_{-0.09+0.19}$
$V_1^{B_s K_{1B}}$	$0.15^{+0.01+0.01}_{-0.01-0.02}$	$1.34^{+0.05-0.06}_{-0.05+0.07}$	$0.76^{+0.08-0.10}_{-0.10+0.14}$	$V_2^{B_s K_{1B}}$	$-0.06^{+0.01-0.01}_{-0.01+0.01}$	$1.65^{+0.02-0.03}_{-0.02+0.04}$	$1.16^{+0.06-0.13}_{-0.06+0.13}$
$A^{B_s h_{1s}}$	$0.09^{+0.00+0.00}_{-0.00-0.00}$	$1.95^{+0.04-0.01}_{-0.04+0.01}$	$2.11^{+0.16-0.07}_{-0.18+0.07}$	$V_0^{B_s h_{1s}}$	$0.51^{+0.01+0.02}_{-0.01-0.02}$	$1.60^{+0.03-0.01}_{-0.03+0.01}$	$1.05^{+0.06-0.04}_{-0.07+0.04}$
$V_1^{B_s h_{1s}}$	$0.17^{+0.01+0.00}_{-0.01-0.00}$	$1.16^{+0.05-0.02}_{-0.05+0.02}$	$0.56^{+0.06-0.03}_{-0.07+0.03}$	$V_2^{B_s h_{1s}}$	$-0.10^{+0.01-0.00}_{-0.01+0.00}$	$1.52^{+0.02-0.01}_{-0.02+0.01}$	$0.95^{+0.03-0.03}_{-0.05+0.03}$

of the form factors, and

- Minor changes occur in $B \rightarrow D$ form factors from their previous values given in the earlier work [1], however, slope parameters show significant difference.
- To determine the physical form factors for $B \rightarrow D_1$ transitions, one may need the mixing angle between $D_1^{1/2}$ and $D_1^{3/2}$ states. A mixing angle $\theta_{D_1} = (5.76 \pm 2.4)^\circ$ is obtained by Belle through a detailed $B \rightarrow D^* \pi \pi$ analysis [71], while $\theta_{D_{s1}} \approx 7^\circ$ is determined from the quark potential model [36].

VI. SUMMARY AND CONCLUSIONS

In this work, we have studied the decay constants and form factors of the ground-state s -wave and low-lying p -wave mesons within a covariant light-front (CLF) approach. In the previous work [1], main ingredients of the CLF quark model were explicitly worked out for both s -wave and p -wave mesons. Besides that various form factors of the D and B mesons, appearing in their transitions

TABLE X: Form factors of $B \rightarrow D_1^{1/2}, D_1^{3/2}$ transitions obtained in the covariant light-front model are fitted to the 3-parameter form Eq. (4.15). All the form factors are dimensionless.

F	$F(0)$	a	b
$A^{BD_1^{1/2}}$	$-0.13^{+0.01-0.02}_{-0.01+0.02}$	$0.85^{+0.09+0.10}_{+0.08-0.18}$	$0.12^{+0.01-0.01}_{+0.02+0.03}$
$V_0^{BD_1^{1/2}}$	$0.11^{+0.01+0.03}_{+0.01-0.03}$	$1.08^{+0.02-0.02}_{+0.02-0.07}$	$0.08^{+0.03+0.02}_{+0.03-0.04}$
$V_1^{BD_1^{1/2}}$	$-0.19^{+0.02-0.01}_{-0.02+0.01}$	$-1.37^{+0.09-0.01}_{+0.08-0.00}$	$1.07^{+0.07+0.01}_{-0.06-0.01}$
$V_2^{BD_1^{1/2}}$	$-0.14^{+0.02-0.02}_{-0.02+0.02}$	$0.84^{+0.11+0.13}_{+0.09-0.21}$	$0.13^{+0.01-0.01}_{+0.02+0.04}$
$A^{BD_1^{3/2}}$	$0.25^{+0.01+0.02}_{+0.01-0.02}$	$1.17^{+0.03+0.01}_{+0.03-0.03}$	$0.33^{+0.02-0.01}_{+0.02+0.01}$
$V_0^{BD_1^{3/2}}$	$0.52^{+0.01+0.04}_{+0.01-0.05}$	$1.14^{+0.04+0.02}_{+0.03-0.06}$	$0.34^{+0.02-0.01}_{+0.02+0.01}$
$V_1^{BD_1^{3/2}}$	$0.58^{+0.01+0.02}_{+0.01-0.03}$	$-0.25^{+0.01-0.03}_{+0.01+0.02}$	$0.29^{+0.00+0.01}_{+0.01-0.01}$
$V_2^{BD_1^{3/2}}$	$-0.10^{+0.01-0.02}_{-0.01+0.04}$	$-5.95^{+2.07+3.80}_{+1.45-14.67}$	$26.2^{+5.8-4.1}_{-11.5+41.0}$
$A^{B_s D_{s1}^{1/2}}$	$-0.17^{+0.02-0.02}_{-0.02+0.02}$	$0.97^{+0.10+0.06}_{+0.10-0.10}$	$0.37^{+0.05-0.04}_{+0.06+0.05}$
$V_0^{B_s D_{s1}^{1/2}}$	$0.13^{+0.01+0.03}_{+0.02-0.03}$	$1.14^{+0.04-0.02}_{+0.04-0.05}$	$0.29^{+0.04-0.03}_{+0.05+0.04}$
$V_1^{B_s D_{s1}^{1/2}}$	$-0.25^{+0.03-0.01}_{-0.03+0.01}$	$-1.20^{+0.10-0.06}_{+0.10+0.07}$	$1.02^{+0.07+0.04}_{-0.06-0.05}$
$V_2^{B_s D_{s1}^{1/2}}$	$-0.17^{+0.02-0.02}_{-0.02+0.02}$	$0.96^{+0.12+0.08}_{+0.11-0.12}$	$0.39^{+0.04-0.04}_{+0.06+0.06}$
$A^{B_s D_{s1}^{3/2}}$	$0.24^{+0.01+0.02}_{+0.01-0.02}$	$1.26^{+0.06+0.00}_{+0.06-0.02}$	$0.60^{+0.06-0.06}_{+0.07+0.06}$
$V_0^{B_s D_{s1}^{3/2}}$	$0.49^{+0.02+0.04}_{+0.02-0.05}$	$1.25^{+0.06+0.01}_{+0.05-0.04}$	$0.63^{+0.05-0.06}_{+0.06+0.07}$
$V_1^{B_s D_{s1}^{3/2}}$	$0.57^{+0.01+0.03}_{+0.01-0.04}$	$-0.11^{+0.02-0.04}_{+0.02+0.04}$	$0.32^{+0.01+0.00}_{+0.01+0.01}$
$V_2^{B_s D_{s1}^{3/2}}$	$-0.09^{+0.01-0.02}_{-0.01+0.03}$	$-4.08^{+1.86+2.63}_{+1.24-7.55}$	$21.1^{+5.3-8.3}_{-3.6+21.8}$

to isovector and isospinor s -wave and p -wave mesons, were calculated within the framework of the CLF model. In the present work, we have updated our results for these mesons, and extended the analysis to determine the form factors for D_s and B_s transitions, and also include the flavor-diagonal isoscalar final states. Calculating the decay constants of most of the s -wave mesons and a few axial vector mesons from the available experimental data for various weak or electromagnetic decays, we have fixed the shape parameter β of the respective mesons, which in turn determine the form factors. A few lattice results are also used for this purpose. Errors in the β parameters are fixed from the corresponding experimental errors, otherwise standard 10% uncertainty is assigned to investigate the effects of variation in the β parameter. We have then proceeded to obtain the form factors in the CLF quark model for heavy-to-heavy and heavy-to-light transitions of the charmed and bottom mesons to the pseudoscalar mesons, vector mesons, scalar mesons and axial vector mesons. The q^2 dependence of the form factors, generally assumed to be given by Eq. (4.15), is expressed through the slope parameters, a and b . Their sensitivity to the errors and the assigned uncertainties of the β parameters is investigated separately for the initial and the final mesons.

Our main results are as follows:

- For $P \rightarrow P$ transitions, B_s form factors at $q^2 = 0$ are similar to that of the B meson, as if the spectator quark does not seem to affect them. Particularly, we observe $F^{B_s D_s} = F^{BD}$, $F^{B_s K} \approx F^{B\pi}$, and $F^{B_s \eta_s} \approx F^{B\eta_q}$, where $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$, and η_s is pure $(s\bar{s})$ state. To lesser extent, the charmed mesons also show a similar trend through $F^{D_s K} = F^{D\pi}$ and $F^{D_s \eta_s} \approx F^{DK}$. Heavy-to-light form factors of the bottom mesons are smaller (around 0.3) than that of the charmed mesons, which are around 0.7. The form factor F_0^{PP} generally shows a monopole behavior, and F_1^{PP} acquires a dipole behavior.
- For $P \rightarrow V$ transitions also, we find $F^{B_s D_s^*} \approx F^{BD^*}$, $F^{B_s \phi} \approx F^{B\rho} \approx F^{B\omega}$, $F^{D_s \phi} \approx F^{DK^*}$ and $F^{D_s K^*} \approx F^{D\rho} \approx F^{D\omega}$, where F denotes any of the four form factors, V, A_0, A_1 and A_2 , at $q^2 = 0$. For the bottom mesons, heavy-to-light form factors are smaller (from 0.2 to 0.4) than their heavy-to-heavy ones, which lie between 0.6 to 1. Due to the reliability in fixing the β parameters for the s -wave mesons, the form factors at $q^2 = 0$ hardly show sensitivity to the errors in the β values, though slope parameters (a and b) generally tend to increase (decrease) with decrease (increase) in β for the initial meson as well as the final meson.
- Comparing $P \rightarrow P, V$ form factors obtained here with the results of other works, BSW model [57], the Melikhov-Stech (MS) model [60], QCD sum rule (QSR) [61], light-cone sum rules (LCSR) [62], lattice calculations [63] and perturbative QCD approach [64], it is found that our form factors agree well with the available lattice results, and are most close to that of the MS model, except for the B_s transitions. The LCSR and BSW model results are usually larger for $P \rightarrow V$ form factors for D and B transitions, whereas the QSR and pQCD calculations are generally lower than our results.
- For $P \rightarrow S$ transitions, we have calculated the form factors involving heavy scalar mesons only. These form factors, though are smaller than the corresponding $P \rightarrow P$ form factors, also satisfy $F^{B_s K_0^*} = F^{Ba_0} = F^{Bf_{0q}}$, $F^{B_s f_{0s}} \approx F^{BD_0^*} = F^{BK_0^*}$, and $F^{D_s f_{0s}} \approx F^{Da_0} = F^{Df_{0q}}$.

All the bottom meson form factors, lying between 0.25 and 0.30, are roughly half of that of the charmed mesons, which are around 0.5-0.6. The suppression of the $D_0^{*0}\pi^-$ production relative to $D^0\pi^-$ one clearly favors a smaller $B \rightarrow D_0^*$ form factor relative to the $B \rightarrow D$ one. The form factor F_0^{PS} shows a monopole behavior, and F_1^{PS} has a dipole behavior in general. The $P \rightarrow S$ form factors are found to increase slowly with q^2 compared to the $P \rightarrow P$ ones. For the bottom sector, the slope parameters are larger in magnitude than that for the charm sector. These parameters (except for the case of negative a) show an increase (decrease) with decrease (increase) in β for each of the initial and final mesons.

- For heavy-to-light $P \rightarrow A(1^{++})$ transitions, the form factor $A^{PA}(0)$ for the bottom mesons, generally lying around 0.25, is larger than that for charmed meson transitions for which it lies close to 0.16. Similarly, the form factor $V_2^{PA}(0)$ is < 0.1 for the charm sector, where as it lies around 0.2 for the bottom transitions. In contrast, the form factor $V_1^{PA}(0)$, lying around 0.4 for the bottom sector is significantly smaller than that for the charm sector, where its value lies between 1.4 and 1.8. Also $V_0^{PA}(0)$ for the bottom transitions is roughly half of its value for the charmed meson transitions. We also observe that $V_1^{PA} > V_0^{PA} > A^{PA} > V_2^{PA}$ for the charmed mesons and $V_1^{PA} > A^{PA} > V_2^{PA} > V_0^{PA}$ for the bottom mesons. All the slope parameters, except for V_1^{PA} and V_2^{PA} for the charmed meson transitions, are found to be positive, and for the bottom mesons, their values are significantly larger than that for the charm sector.
- For heavy-to-light transitions $P \rightarrow A(1^{+-})$, the form factors, $A^{PA}(0)$, $V_0^{PA}(0)$ and $V_1^{PA}(0)$, are positive, where as $V_2^{PA}(0)$ is negative and small (around -0.1) for the bottom as well as the charmed sector. These follow the pattern: $V_1^{PA} > V_0^{PA} > A^{PA} > |V_2^{PA}|$ for the charmed mesons and $V_0^{PA} > V_1^{PA} > A^{PA} \geq |V_2^{PA}|$ for the bottom mesons. Numerically, the form factor $A^{PA}(0)$ is generally around 0.1, where as $V_0^{PA}(0)$ lies close to 0.5 for all the cases. Form factor $V_1^{PA}(0)$, lying between 0.15 to 0.20, for the bottom sector is significantly smaller than that for the charm sector, where its value lies between 1.3 to 1.6. Typically for the heavy-to-light $P \rightarrow A(1^{+-})$ transitions, the form factors $A^{PA}(0)$ and $V_1^{PA}(0)$ usually increase (decrease) with increasing (decreasing) beta for initial meson as well as for final meson. Both $V_0^{PA}(0)$ and $V_2^{PA}(0)$ form factors decrease (increase) in magnitude with increasing (decreasing) β for the initial mesons, and show the opposite trend for the final mesons. For the bottom sector, the slope parameters are found to be larger than that for the charm sector.
- For $B \rightarrow D_1^{1/2}/D_1^{3/2}$ transitions, all the form factors lie between 0.1 to 0.2, except for $V_0(0)$ and $V_1(0)$ for the transitions emitting $P_1^{3/2}$ states, for which these lie between 0.5 to 0.6. Slope parameters carry positive values, except a for the V_1 form factor. Reverse changes occur in the form factors due to variation in β values for initial and final mesons, i.e., increase in β for the initial (final) state meson tend to decrease (increase) the magnitude of the form factors.
- Now several model calculations for $B \rightarrow A$ form factors are available: the ISGW2 model [11], the constituent quark-meson model (CQM) [65], the QCD sum rules (QSR) [66], light cone sum rules (LCSR) [67], and the perturbative QCD (pQCD) approach [64]. Significant

differences are observed, since these approaches differ in their treatment of dynamics of the form factors. For instance, $V_0^{Ba_1} = 1.20$, obtained in the CQM model, and 1.01 in the ISGW2 model, is much larger than its values obtained in other approaches. The BaBar and Belle measurements [68, 69] of $\overline{B}^0 \rightarrow a_1^\pm \pi^\mp$ seem to favor a value of $V_0^{Ba_1} \approx 0.30$ [70]. We have earlier pointed out [1] that relativistic effects could manifest in heavy-to-light transitions at maximum recoil where the final-state meson can be highly relativistic, which can naturally be considered in the CLF model. Various form factors, calculated using the CLF model, have earlier been used to study weak hadronic and radiative decays of the bottom mesons emitting p -wave mesons [7, 38], and a good agreement between theory and available experimental data could be obtained. It has been pointed out in the previous work [1] that the requirement of HQS is also satisfied for the decay constants and the form factors obtained in the CLF quark model. Particularly, it has been shown that the Bjorken [72] and Uraltsev [73] sum rules for the Isgur-Wise functions are satisfied.

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